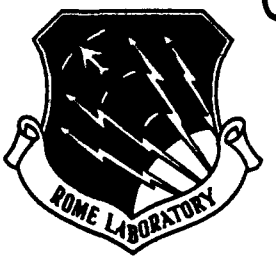
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RL-TR-92-134
In-House Report
June 1992



ANALYTICAL CHARACTERIZATION OF BISTATIC SCATTERING FROM ROUGH SURFACES: DEPENDENCE ON SURFACE CORRELATION FUNCTION

Lisa M. Sharpe

OCT 23 1992

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APPROVED: John F Lennon

JOHN F. LENNON
Chief, Environmental Effects Branch
Electromagnetics & Reliability Directorate

FOR THE COMMANDER: John K Schindler

JOHN K. SCHINDLER, Director
Electromagnetics & Reliability Directorate

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REPORT DOCUMENTATION PAGE

Form Approved
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Public reporting for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	June 1992	In-House 31 March 1992

4. TITLE AND SUBTITLE
Analytical Characterization of Bistatic Scattering from Rough Surfaces: Dependence on Surface Correlation Function

5. FUNDING NUMBERS
PE: 61102F
PR: 2305
TA: 2305J4
WU: 2305J409

6. AUTHOR(S)
Lisa M. Sharpe

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Rome Laboratory/ERC
Hanscom AFB, MA 01731-5000

8. PERFORMING ORGANIZATION REPORT NUMBER
RL-TR-92-134

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release; Distribution Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

Analyses of the statistical properties of electromagnetic scattering from random rough surfaces for finite clutter cells have assumed that the rough surface can be modeled by a Gaussian correlation function. However, some real surfaces are more accurately described by correlation functions other than Gaussian. In this study, the effect of decreasing the clutter cell size is examined for power law and exponential, as well as Gaussian correlation functions for a one dimensionally rough surface. Results obtained in this study show that for a Gaussian correlated surface and a surface described by a power law correlation the scattering statistics are similar and deviate significantly from Rayleigh with decreasing cell size, particularly in the backscatter region. For exponentially correlated surfaces, however, the statistics of the scattering are not changed when the cell size is decreased. This is important because urban areas are described by an exponential correlation function.

14. SUBJECT TERMS			15. NUMBER OF PAGES
Correlation function	Gaussian correlation		54
Rayleigh scattering	Exponential correlation		16. PRICE CODE
Bistatic scattering	Power law correlation	Scattering facets	

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

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ANALYTICAL CHARACTERIZATION OF BISTATIC SCATTERING FROM ROUGH SURFACES: DEPENDENCE ON SURFACE CORRELATION FUNCTION

1. INTRODUCTION

The short pulse width in high resolution radars has the effect of shortening the clutter cell illuminated by the radar. This smaller area would result in smaller clutter returns if the statistical distribution of the power scattered by the clutter does not change as the cell size is decreased. The scattered power from large clutter cells, which have many similar sized scattering facets, has a Rayleigh distribution. A large number of scattering facets is a requirement for Rayleigh scattering. However, as the cell size is decreased, the number of scattering facets is decreased, and Rayleigh scattering may no longer result. In this report, we will see what effect decreasing the clutter cell size has on several different types of surfaces.

In studies by Papa and Woodworth[1] and Sharpe[2] it is shown that decreasing the size of the clutter cell can significantly affect the statistical distribution of the scattered power. These studies have only examined surfaces which are described by a Gaussian correlation function. In this report other surfaces that are described by a power law correlation function and an

Received for Publication 12 June 1992

[1] Papa, R.J. and Woodworth, M.B. (1991) *The Mean and Variance of Diffuse Scattered Power as a Function of Clutter Resolution Cell Size*, RADC-TR-91-09.

[2] Sharpe, L.M. (1991) *Analytical Characterization of Bistatic Scattering from Gaussian Distributed Surfaces*, RL-TR-91-351.

exponential correlation function are studied. It is found that the type of surface, whether it is smoothly varying or very jagged, has a significant impact on the distribution of the scattered power.

The first section of this report will give a brief description of the correlation functions that are used, as well as the type of surfaces described by each one. Results for the different correlation functions will then be presented and compared. Finally, some conclusions about the effect of cell size and correlation functions on clutter statistics will be made.

2. CORRELATION FUNCTIONS

A complete statistical description of a random rough surface involves the correlation function of the surface as well as the height distributions. The height distribution functions at each point, which describe the height deviations from a certain mean level, are assumed here to be Gaussian with a mean level of zero. The correlation function, $C(\tau)$, is a function of the separation of two points on the surface and is a measure of how closely related the distributions at these two points are. The separation parameter, τ, is given by $\tau = (x_1 - x_2)$, with x_1 and x_2 being points on the surface. When $C(\tau) \cong 1$, the heights, ζ_1 and ζ_2 at x_1 and x_2 are fairly strongly correlated so that knowledge of the height at one point gives information about the height at the other point, and when $C(\tau) \cong 0$, the heights are uncorrelated, so that knowledge of the height at one point tells nothing of the height at the other point. For a purely random surface, $C(\tau)$ is an even function and:

$$\lim_{\tau \to 0} C(\tau) = 1, \quad \lim_{\tau \to \infty} C(\tau) = 0 . \tag{1}$$

The correlation distance, T, is defined to be the distance in which $C(\tau)$ drops to the value e^{-1}. It is this correlation distance that determines whether a clutter cell is large or small. For "infinite" clutter cells, $T << L$ where L is the dimension over which the scattering occurs. The total scattered power divided by L is the average scattered power. However, in the finite cell size studies done here, the cell size ranges from $L=2T$ to $L=12T$, so the conditions necessary for Rayleigh scattering are violated to different extents.

The correlation functions used in this study are the Gaussian, Power Law, and Exponential. They are given by:

$$C_g(\tau) = \exp(-\tau^2/T^2) \qquad \text{Gaussian} \tag{2}$$

$$C_{pl}(\tau) = [1 + (\tau/T)^2]^{-3/2} \qquad \text{Power Law} \qquad (3)$$

$$C_{exp}(\tau) = \exp(-|\tau|/T) \qquad \text{Exponential} \qquad (4)$$

The Gaussian correlation is used to describe smoothly curving surfaces having height derivatives at all points.[3] The power law correlation represents surfaces that are somewhat less correlated than the Gaussian surface. Asphalt roads and gravel surfaces are typically described by a power law correlation function.[4] Barrick and Peake[5] found a power law correlation empirically fitted measured points of an asphalt surface. Surfaces described by an exponential correlation function are jagged and have many vertical facets, such as urban areas having buildings and houses as the scattering surfaces. The three different correlation functions are plotted in Figure 1. The correlation distance is T=1 for all three cases. Note that at τ=1, all three functions cross the e^{-1} point. Surfaces described by these different correlation functions are quite different and can have quite different scattering statistics.

Figures 2 through 7 show two different realizations for surfaces with the three correlation functions studied. The rms height of these surfaces is σ=0.5m, which is obviously not a realistic value for asphalt surfaces, where one would expect $\sigma << 0.5$m, or urban areas where $\sigma >> 0.5$m, but for comparing the different correlation functions, a common value for σ was used. To determine these sample surfaces a random set of uncorrelated Gaussian heights was generated and multiplied by the appropriate correlation matrix. The generation of the correlation matrix and correlated surfaces is discussed in Appendix A. The result will be the correlated surface heights. For Figures 2, 3 and 4, the same set of random numbers was used to generate the surfaces. In comparing these three figures it is obvious that the exponential surface is much more jagged and has many more scattering facets than the Gaussian correlated surface. The surface with a power law correlation is not as jagged as the exponential and not as smooth as the Gaussian surface. Even though the surfaces are all quite different, the same large scale structure can be seen in all three because the same set of random numbers was used to generate all three surfaces. The surfaces shown in Figures 5, 6 and 7 were generated by different sets of random numbers. Again we see that the Gaussian correlated surface is fairly smooth, the exponential surface is quite jagged, and the power law correlated surface lies somewhere between the other two in terms of irregularities. Because these three surfaces were generated by different sets of random numbers, the large scale structure is not common to the three surfaces.

---

[3] Ruck, G.T., Barrick, D.E., Stuart, W.D., and Krichbaum, C.K. (1970) *Radar Cross Section Handbook*, **2**, Plenum Press, New York.

[4] Cohen, E., Bell, M., Lin, C., Holst, J., and Tomasanis, D. (1988) *Mathematical Analysis and Programming for Electromagnetic Applications*, RADC-TR-88-18, ADB124496.

[5] Barrick, D.E., and Peake, W.H. (1967) *Scattering from Surfaces with Different Roughness Scales: Analysis and Interpretation*, BAT-197A-10-3.

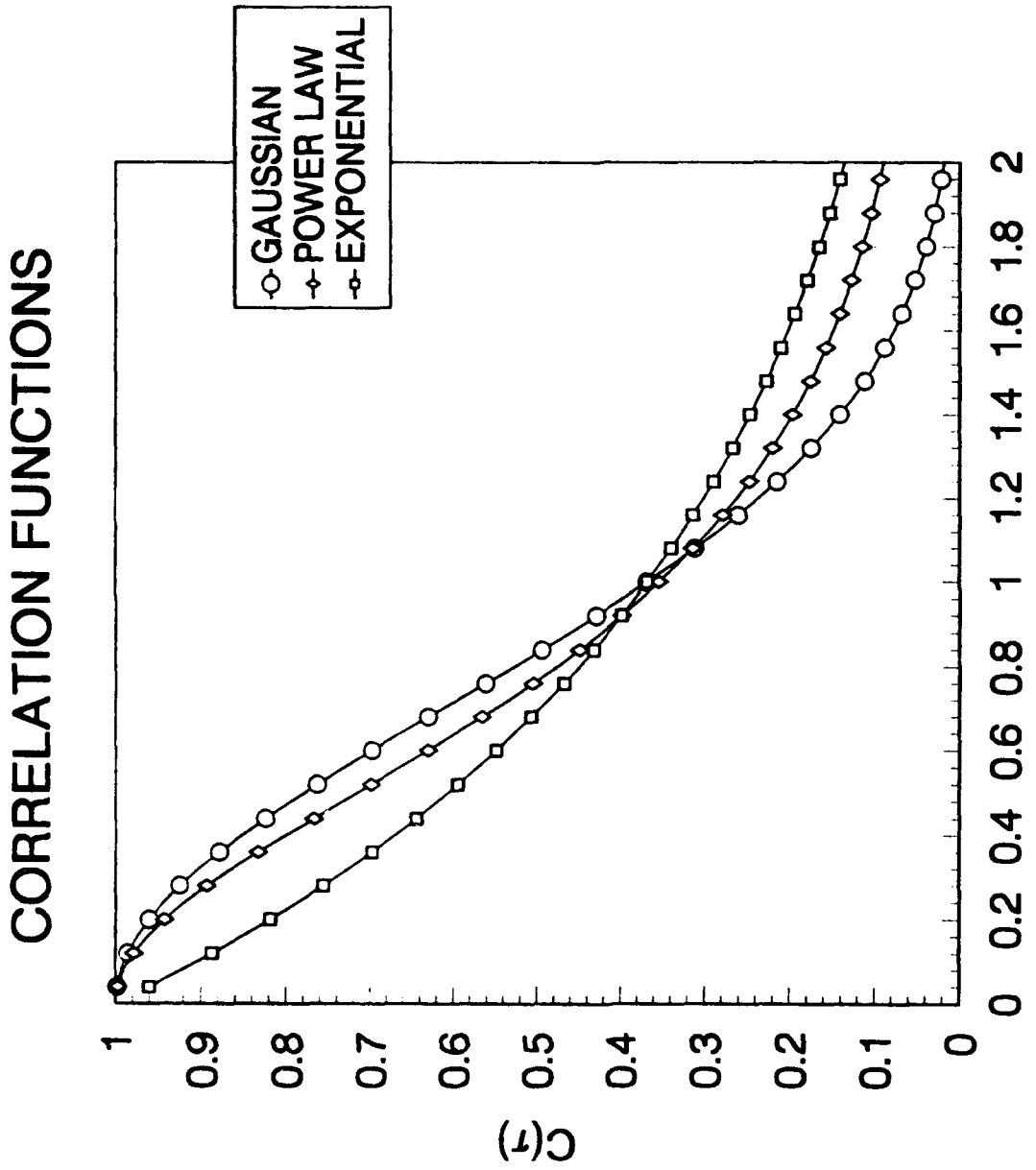


Figure 1. Correlation Functions

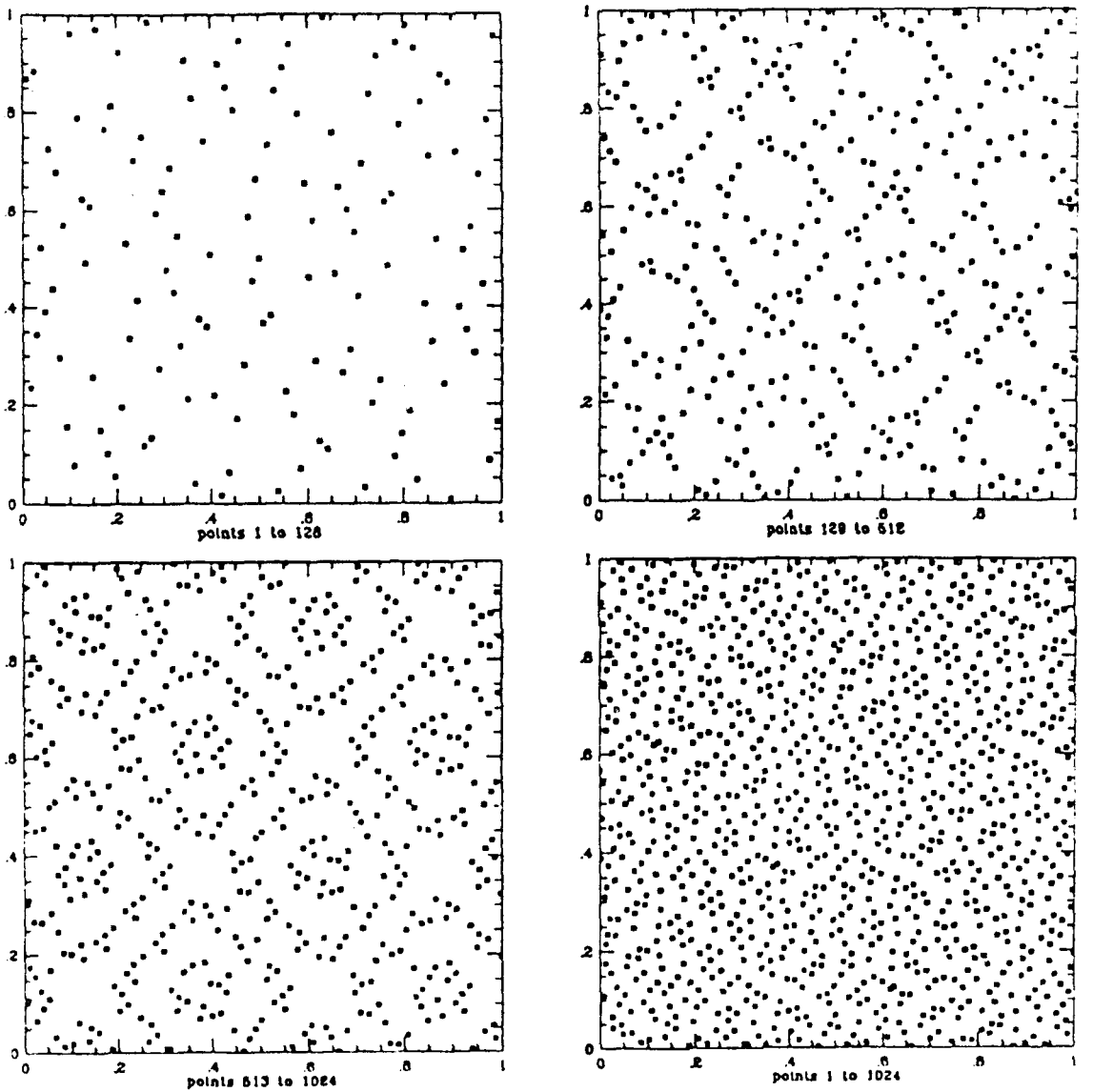


Figure 2. Integration Area Being Filled by a Sobol Sequence

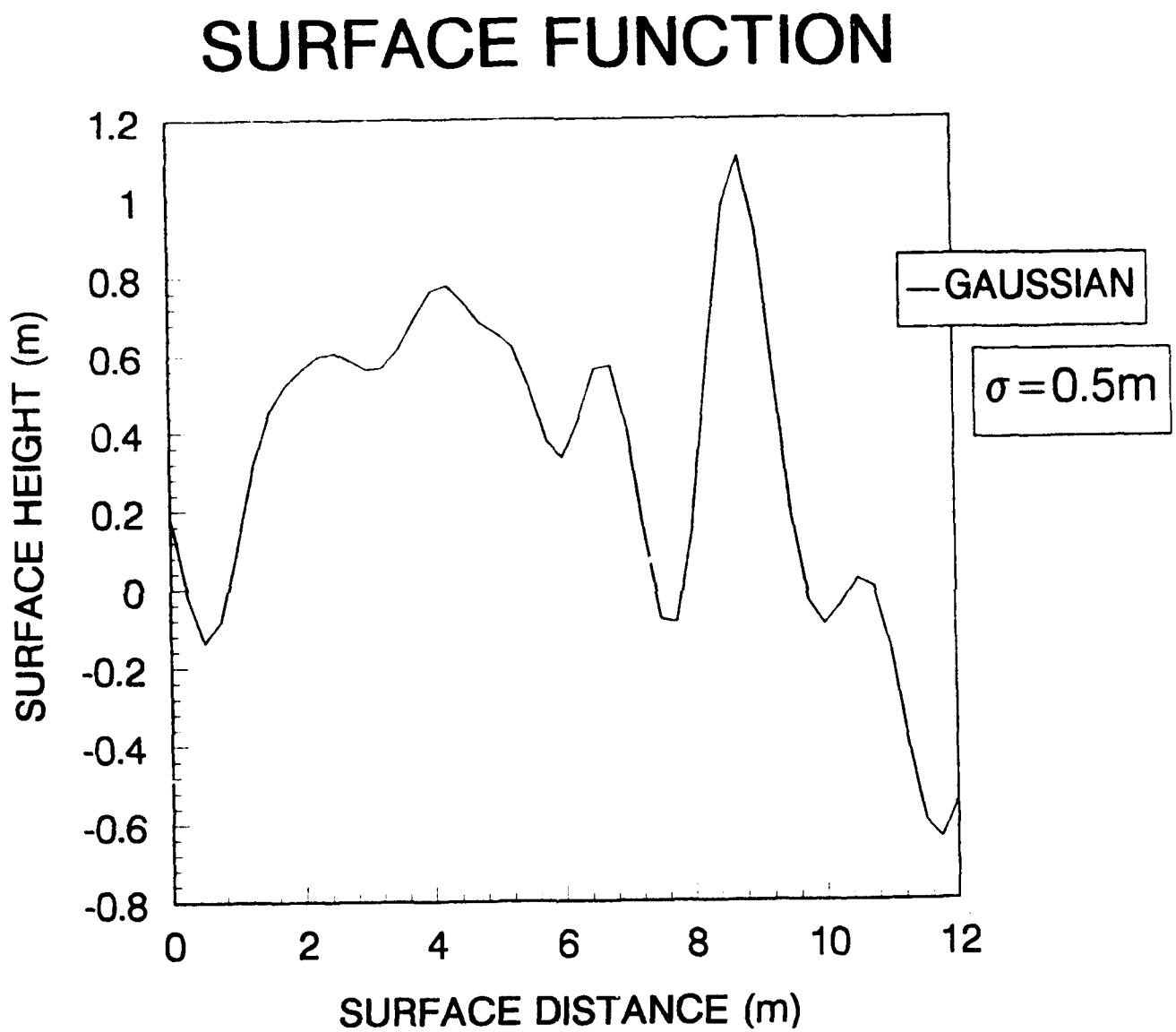


Figure 3. Gaussian Correlation Surface

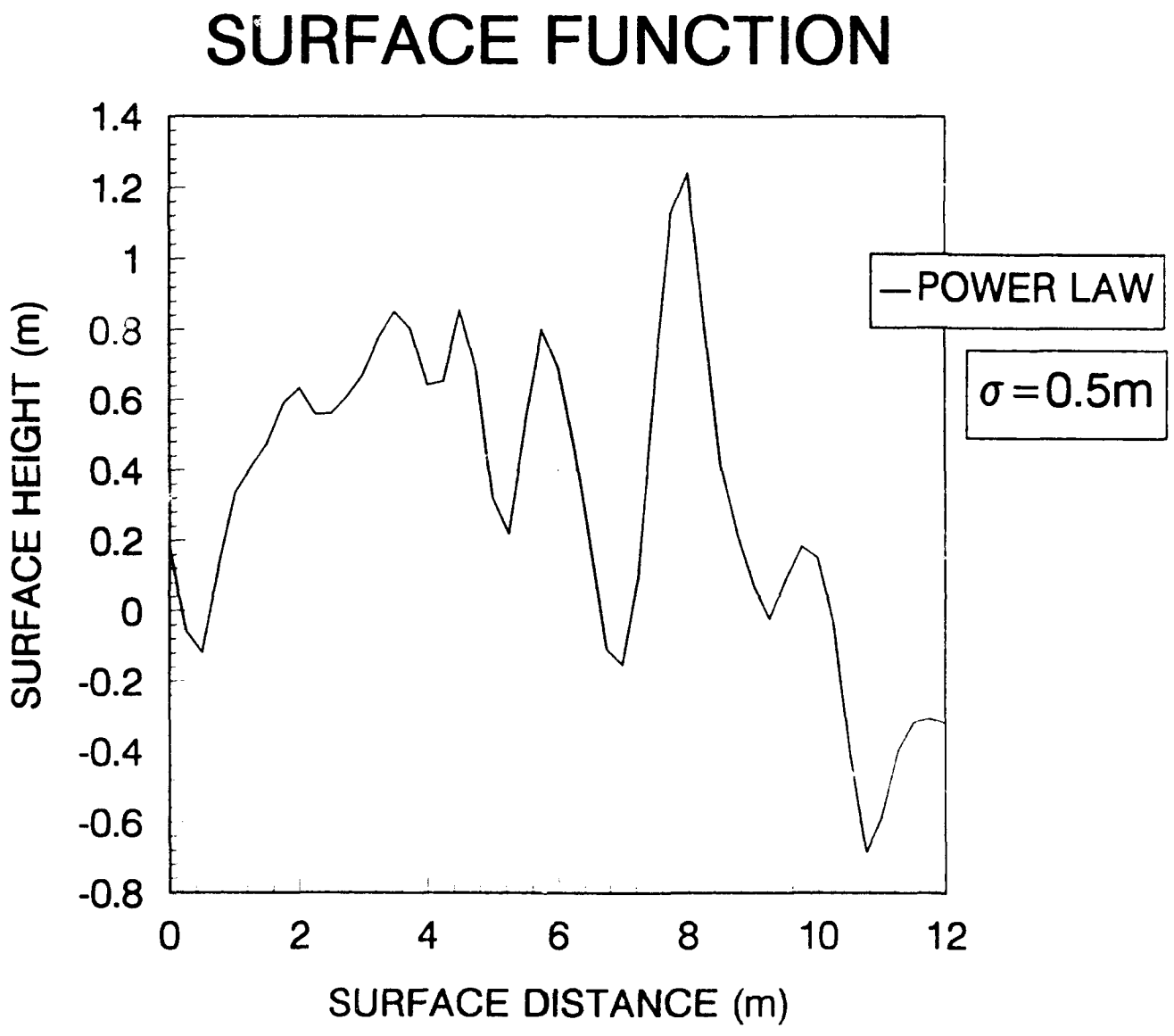


Figure 4. Power Law Correlation Surface

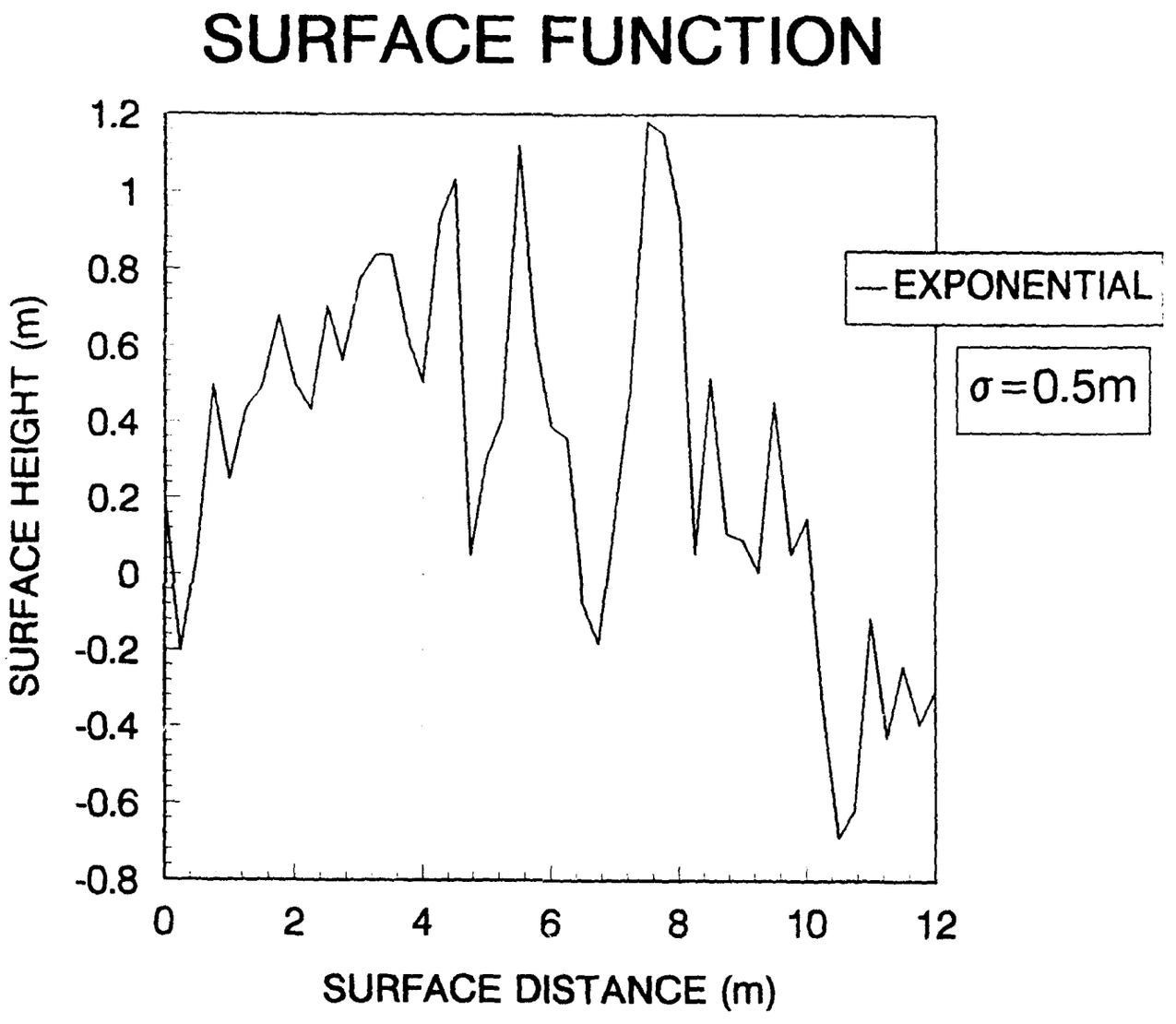


Figure 5. Exponential Correlated Surface

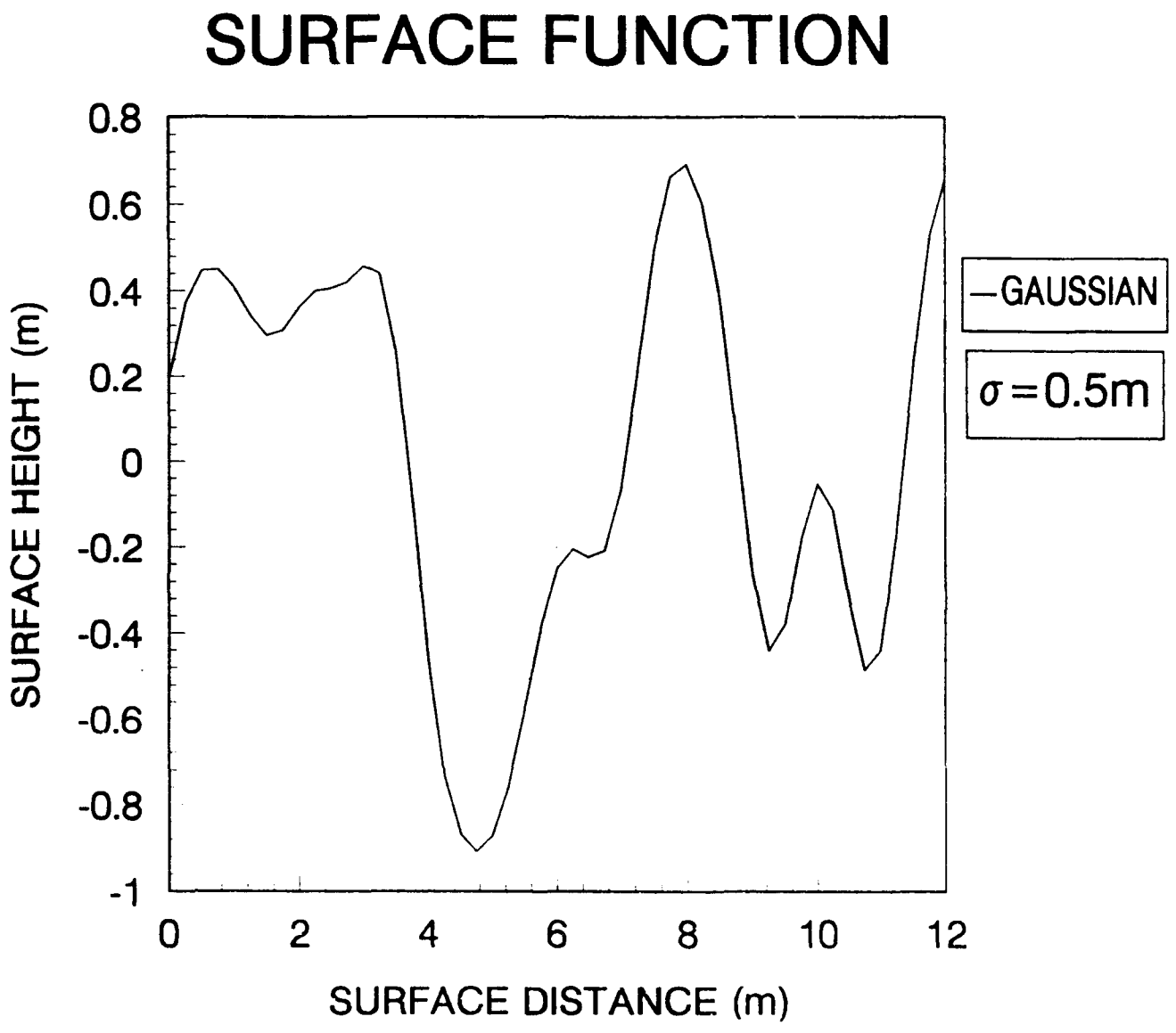


Figure 6. Gaussian Correlated Surface

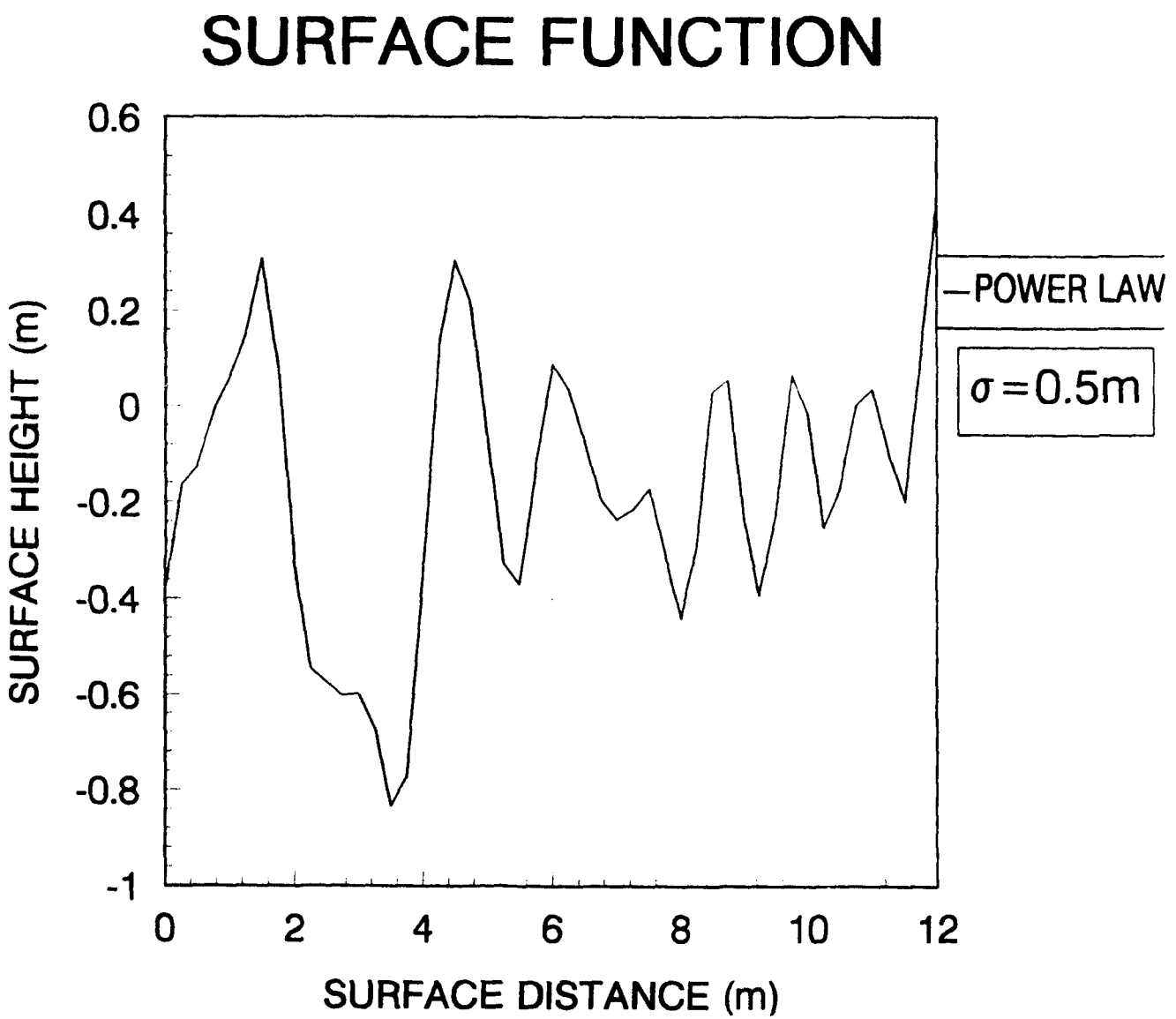


Figure 7. Power Law Correlated Surface

3. ONE-DIMENSIONAL SCATTERING MODEL

Scattering from large clutter cells is typically assumed to have a Rayleigh distribution. The equation for a Rayleigh distribution is:

$$p(x) = \frac{1}{\mu} \exp(-x/\mu),$$

where x is the power. The mean is μ, and the variance is given as $\sigma^{00} = \overline{(x^2)} - (\bar{x})^2$, where $\bar{x}$ is the mean, and $\overline{(x^2)}$ is the second moment given by:

$$\overline{(x^2)} = \int_0^\infty \frac{x^2}{\mu} \exp\left(-\frac{x}{\mu}\right) dx = 2\mu^2 . \tag{5}$$

The variance is therefore given as $\sigma^{00} = 2\mu^2 - \mu^2 = \mu^2$, and the relationship between the mean and the variance of the scattered power for Rayleigh scattering is $\sigma^{00} = (\sigma^0)^2$. However, as the cell size is decreased, violating the condition $T \ll L$, this relationship may no longer be valid. In previous studies, the mean value of the scattered power for a finite clutter cell of a one dimensionally rough surface was given as[1,2]:

$$\sigma^0 = \left[\frac{\pi F^2}{2L\lambda}\right] \int_{-L/2}^{L/2} dx_1 \int_{-L/2}^{L/2} dx_2 \left[\chi_2 - \chi_1 \chi_1^*\right] \cos(v_x \tau) \tag{6}$$

where:

τ = $x_1 - x_2$

χ_2 = $\exp[-\Sigma^2 (1-C(\tau)]$ is the characteristic function for bivariate height distribution

χ_1 = $\exp(-\Sigma^2/2)$ is the characteristic function for univariate height distribution

Σ^2 = $\sigma^2 v_z^2$ = Rayleigh parameter squared

σ = rms surface height

v_x = $(2\pi/\lambda)[\sin(\theta_i)-\sin(\theta_s)]$

v_z = $-(2\pi/\lambda)[\cos(\theta_i)+\cos(\theta_s)]$

$$F = \frac{2R\left(1 + \cos\left(\phi_i + \phi_s\right)\right)}{\cos\phi_i + \cos\phi_s}.$$

However, performing a change of variables from x_1 and x_2 to τ and σ makes it possible to solve the integral over σ analytically, leaving a single integral for the mean value of the scattered power given by (See Appendix B):

$$\sigma^0 = \frac{\pi F^2}{L\lambda} \int_{\phi}^{L} d\tau \left[L-\tau\right] \chi_2 - \chi_1\chi_1^* \cos\left(v_x\tau\right). \tag{7}$$

The variance for the finite clutter cell is calculated from:

$$\sigma^{00} = \left(\frac{\pi F^2}{2L\lambda}\right) \int_{L/2}^{L/2}\int\int\int dx_1 dx_2 dx_3 dx_4 \cos\left(v_x\left(x_1 - x_2 + x_3 - x_4\right)\right) + \left[\chi_4 - \chi_{12}\,\chi_{34}\right] \tag{8}$$

where

χ_{12} = $\exp[-\Sigma^2 (1-C_{12})]$

χ_{34} = $\exp[-\Sigma^2 (1-C_{34})]$

C_{ij} = $C(\tau)$ as defined in Eq. (2), (3), or (4).

The calculated variance is compared to the Rayleigh value of the variance, $\sigma^{00} = (\sigma^0)^2$ to determine if decreasing the cell size will cause the scattering to become non-Rayleigh. Note that the surface correlation function is used in the calculation of both the mean and the variance of the scattered power.

Because of the intensive calculations done in this study, particularly for the variance, efficient numerical integration techniques were required. The integration scheme used is a Sobol sequence.[6] It is considerably more efficient than conventional integration schemes. In this method, the n-space that is being integrated over is filled by quasi-random points that are "maximally avoiding" each other, and therefore fill the space more uniformly than a purely random sequence. Figure 8 shows a two-dimensional space being filled by Sobol sequences. The algorithm used for Sobol sequence integration as well as a description of the technique is given in Reference 5.

4. GAUSSIAN AND POWER LAW CORRELATION FUNCTION RESULTS

The normalized mean and variance of the scattered power for the Gaussian correlated surfaces are shown in Figures 9 through 16. The curve with the small boxes represents the mean value of the scattering σ^0, the curve with the triangles represents the Rayleigh value of the variance $(\sigma^0)^2$, and the curve with the large boxes represents the calculated variance for the finite clutter cell. In Figures 9, 10, and 11, θ_i=75°, λ=0.25m, σ=0.5, and L decreases from 12m in Figure 9, to 6m in Figure 10, and to 3m in Figure 11. It is clear that as the cell size decreases, the scattering becomes less Rayleigh, particularly in the backscatter region, due to fewer scattering facets oriented in this direction. This means that decreasing the cell size for this type of surface will have more severe consequences for a monostatic radar than for a bistatic radar.

Figure 12 is the same as Figure 11 except that the wavelength is decreased to λ=0.1m. Comparing Figures 11 and 12 shows that the scattering is closer to Rayleigh for the longer wavelength. This is because there are more scatterers per wavelength as the wavelength is increased.

Figure 13 is the same as Figure 12 except the surface roughness is decreased to σ^0=0.2m. For the smoother surface the scattering is concentrated more in the specular direction while the rougher surface scatters more diffusely.

For Figures 14 through 16 the angle of incidence is θ_i=30°. These results show the same trends as seen for the θ_i=75° case, that is, for the longer wavelength the scattering is closer to Rayleigh, and the scattering from the rougher surface is more diffuse over the scattering region.

[6] Press, W.H. and Teukolsky, S.A. (1989) Quasi- (that is sub -) random numbers. *Computers in Physics*: 76-79.

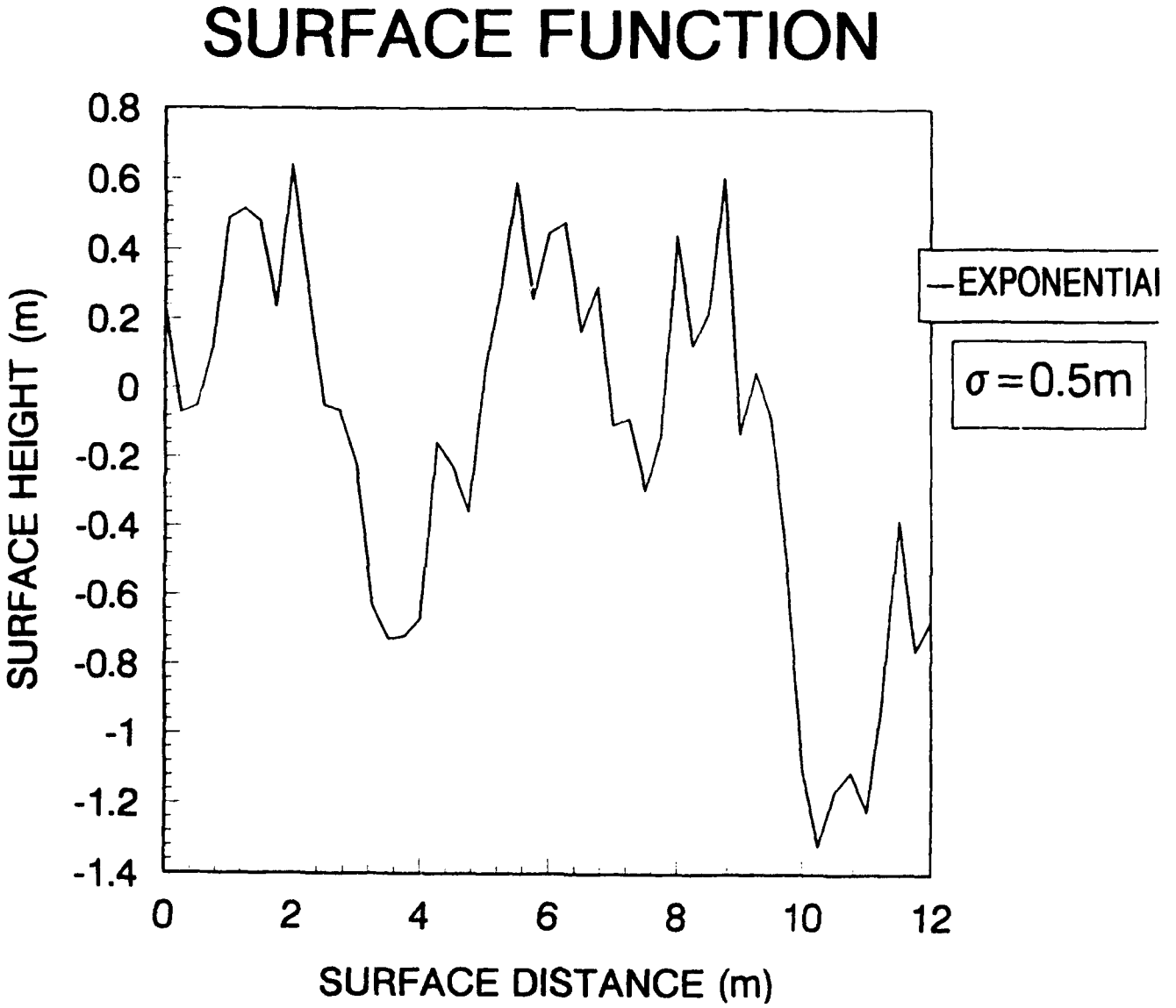


Figure 8. Exponential Correlated Surface

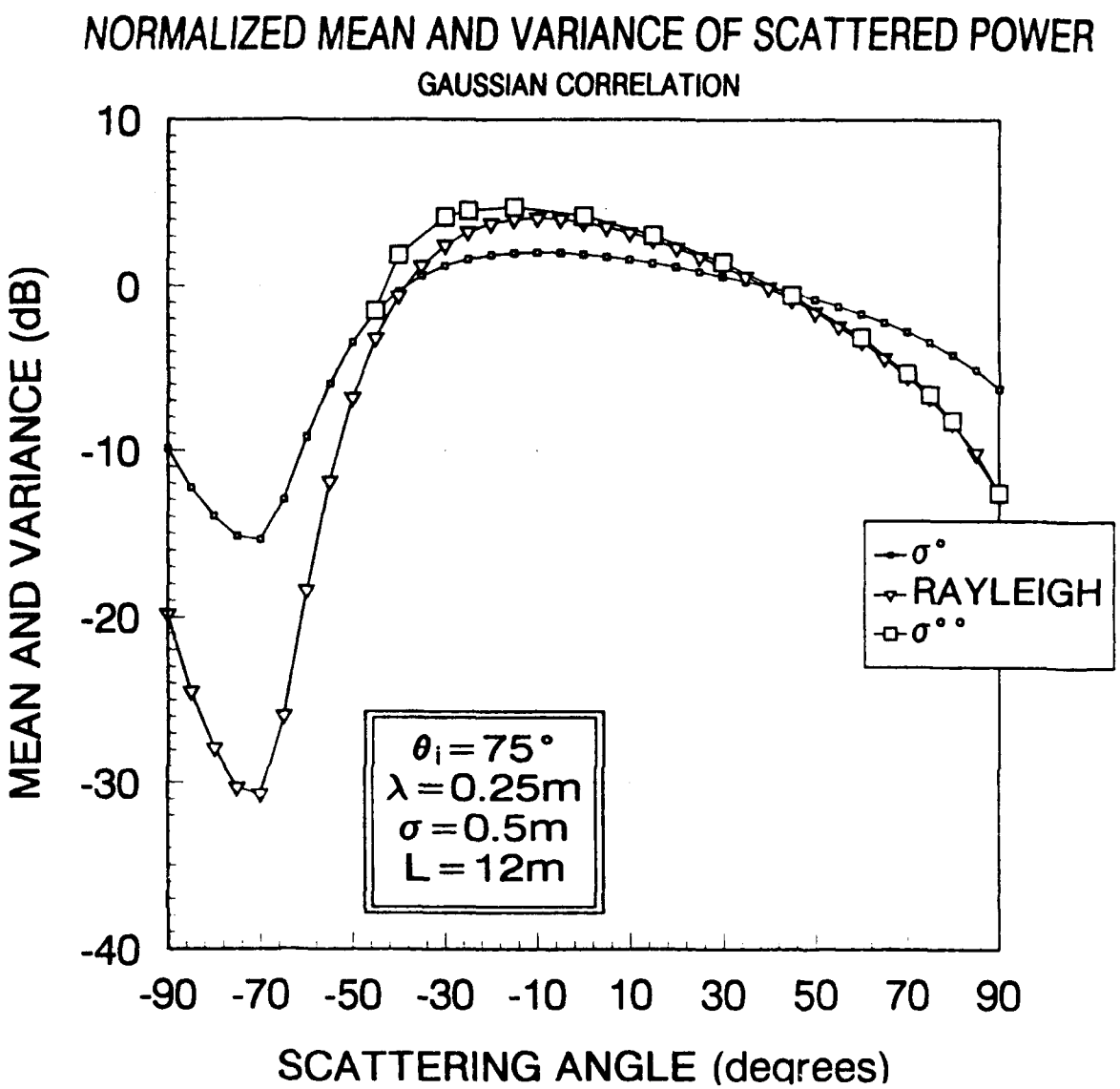


Figure 9. Normalized Mean and Variance for Gaussian Surface, $\theta_i = 75°$, $\lambda = 0.25$m, $\sigma = 0.5$m, L = 12m

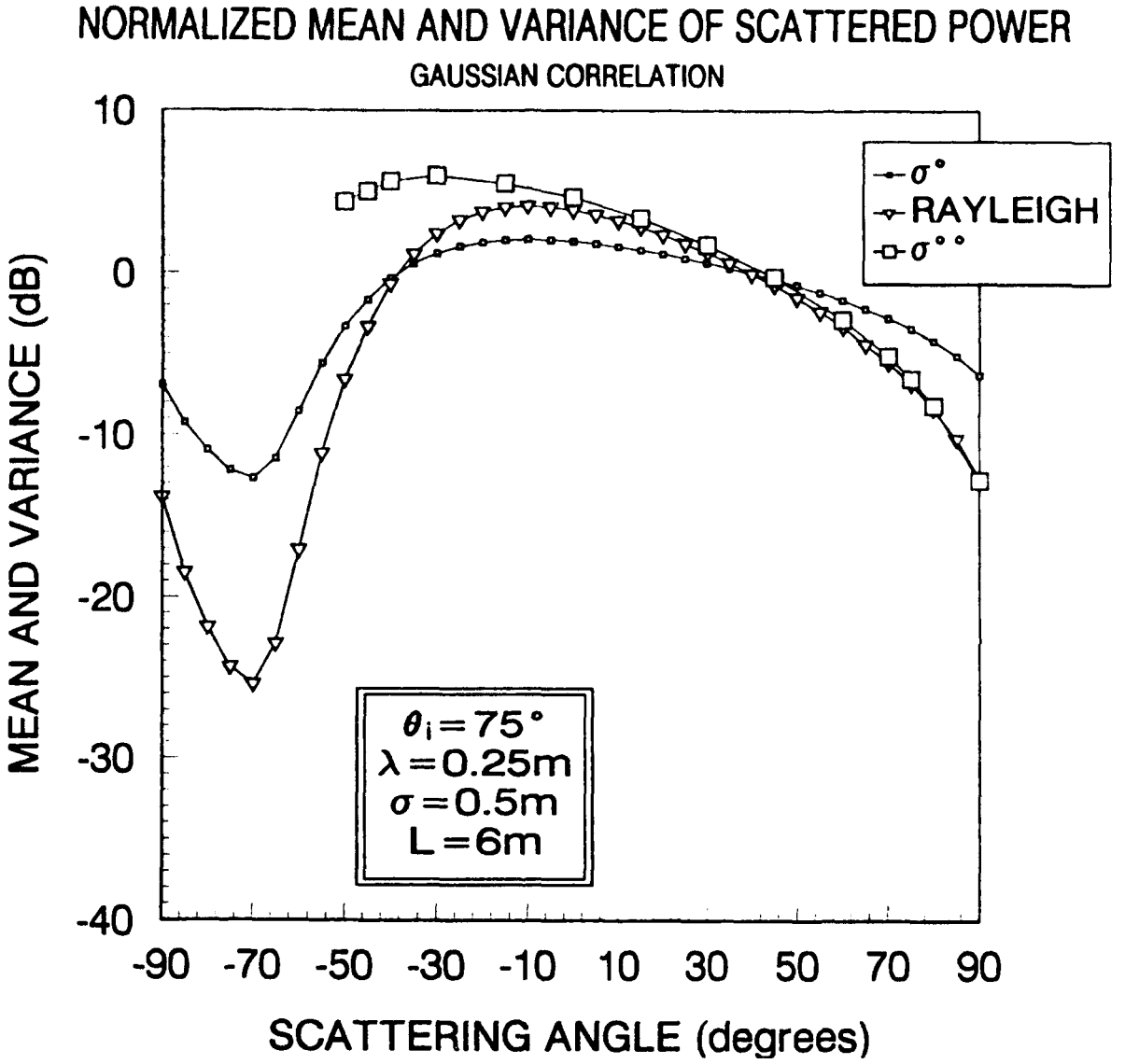


Figure 10. Normalized Mean and Variance for Gaussian Surface, $\theta_i = 75°$, $\lambda = 0.25m$, $\sigma = 0.5m$, $L = 6m$

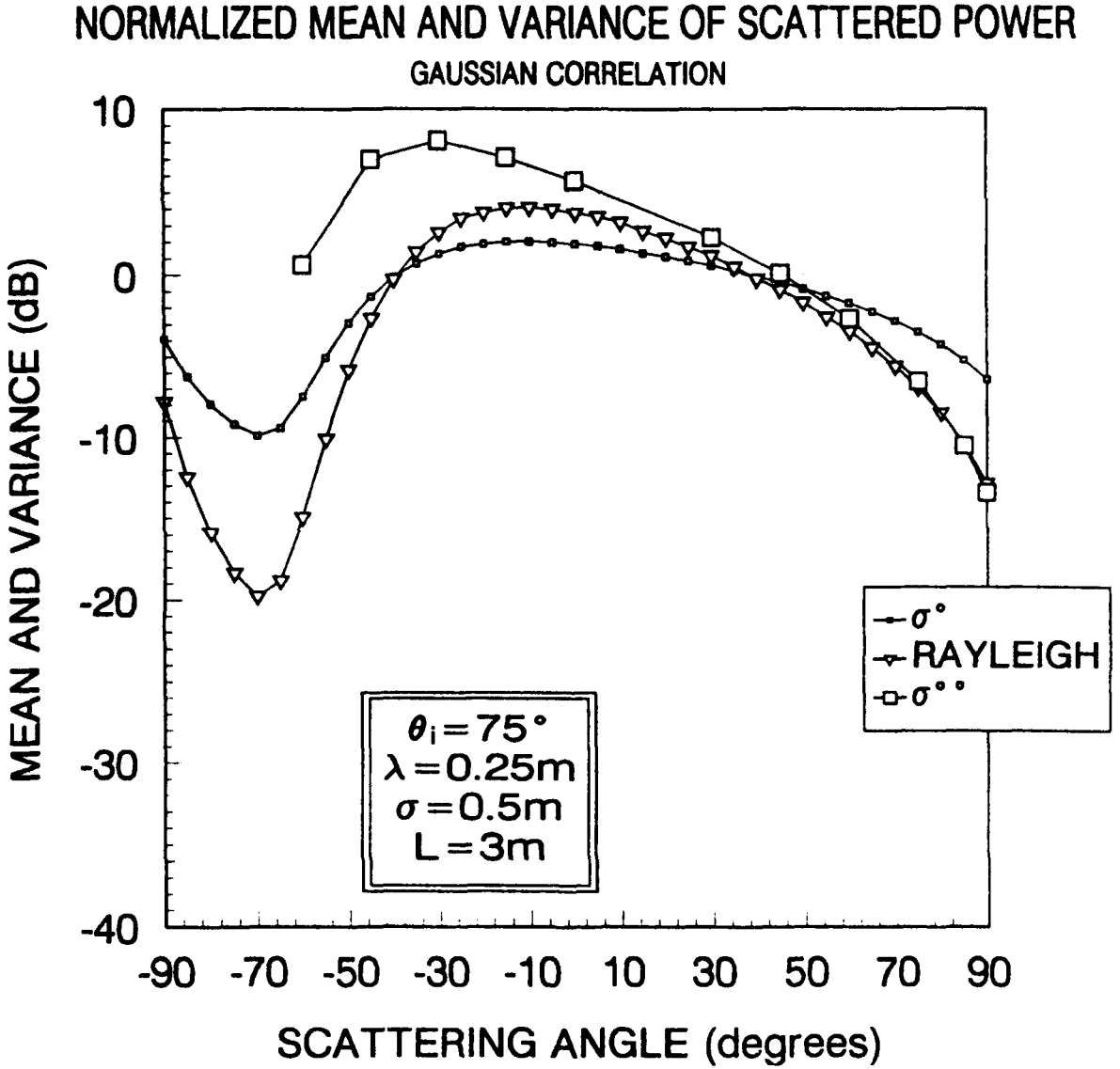


Figure 11. Normalized Mean and Variance for Gaussian Surface. θ_i = 75°, λ = 0.25m, σ = 0.5m, L = 3m

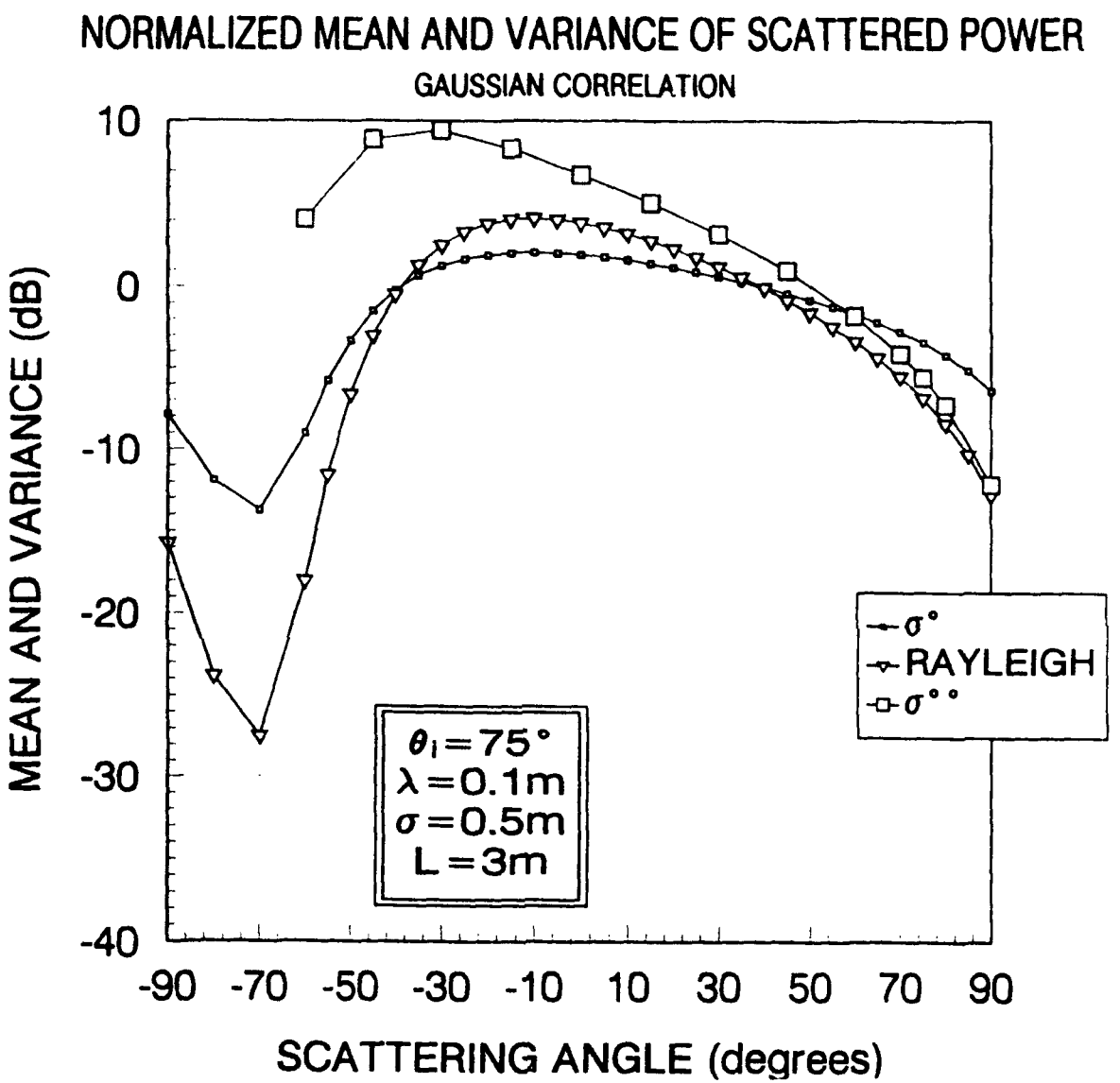


Figure 12. Normalized Mean and Variance for Gaussian Surface, $\theta_i = 75°$, $\lambda = 0.1$m, $\sigma = 0.5$m, $L = 3$m

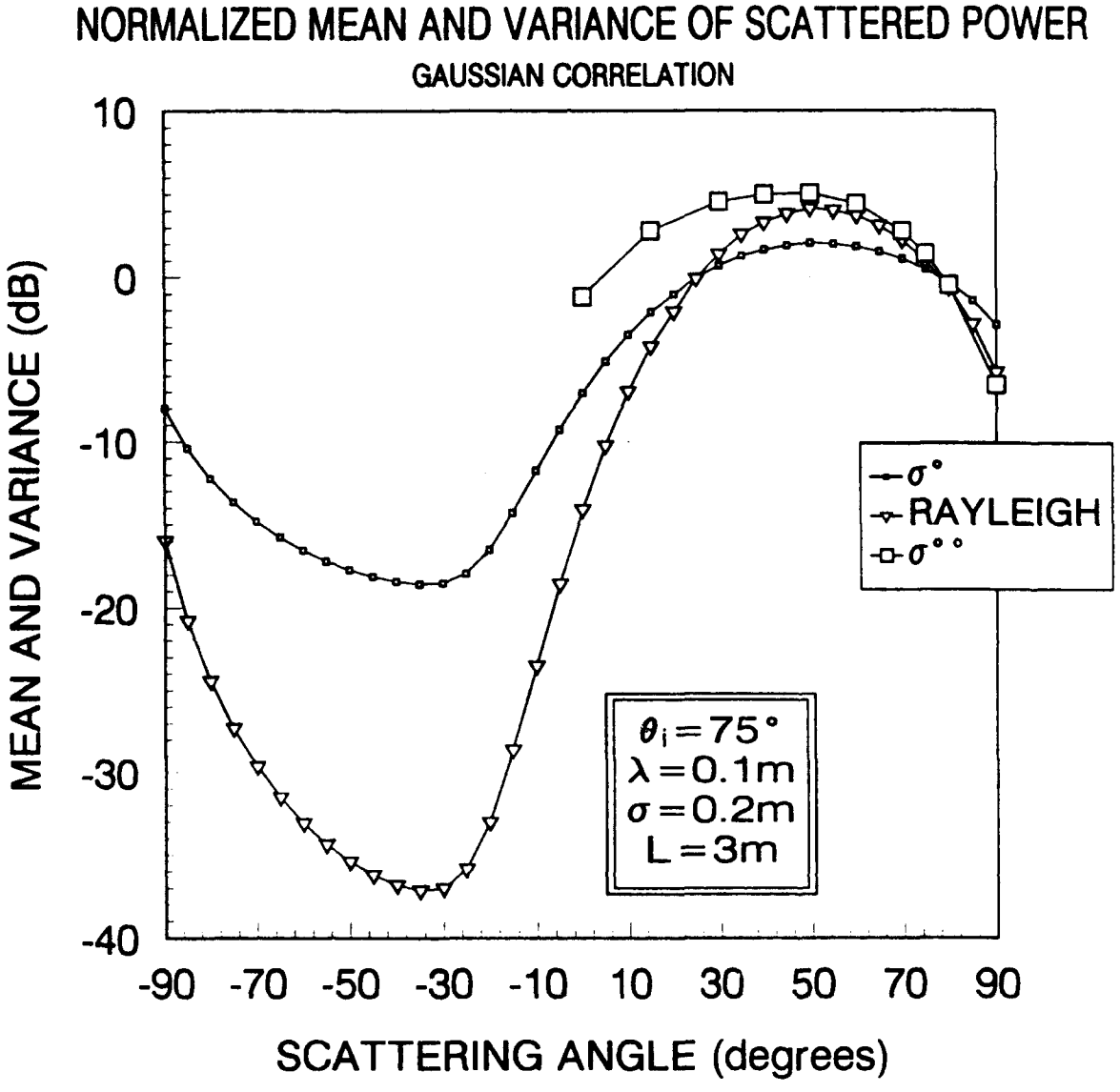


Figure 13. Normalized Mean and Variance for Gaussian Surface, θ_i = 75°, λ = 0.1m, σ = 0.2m, L = 3m

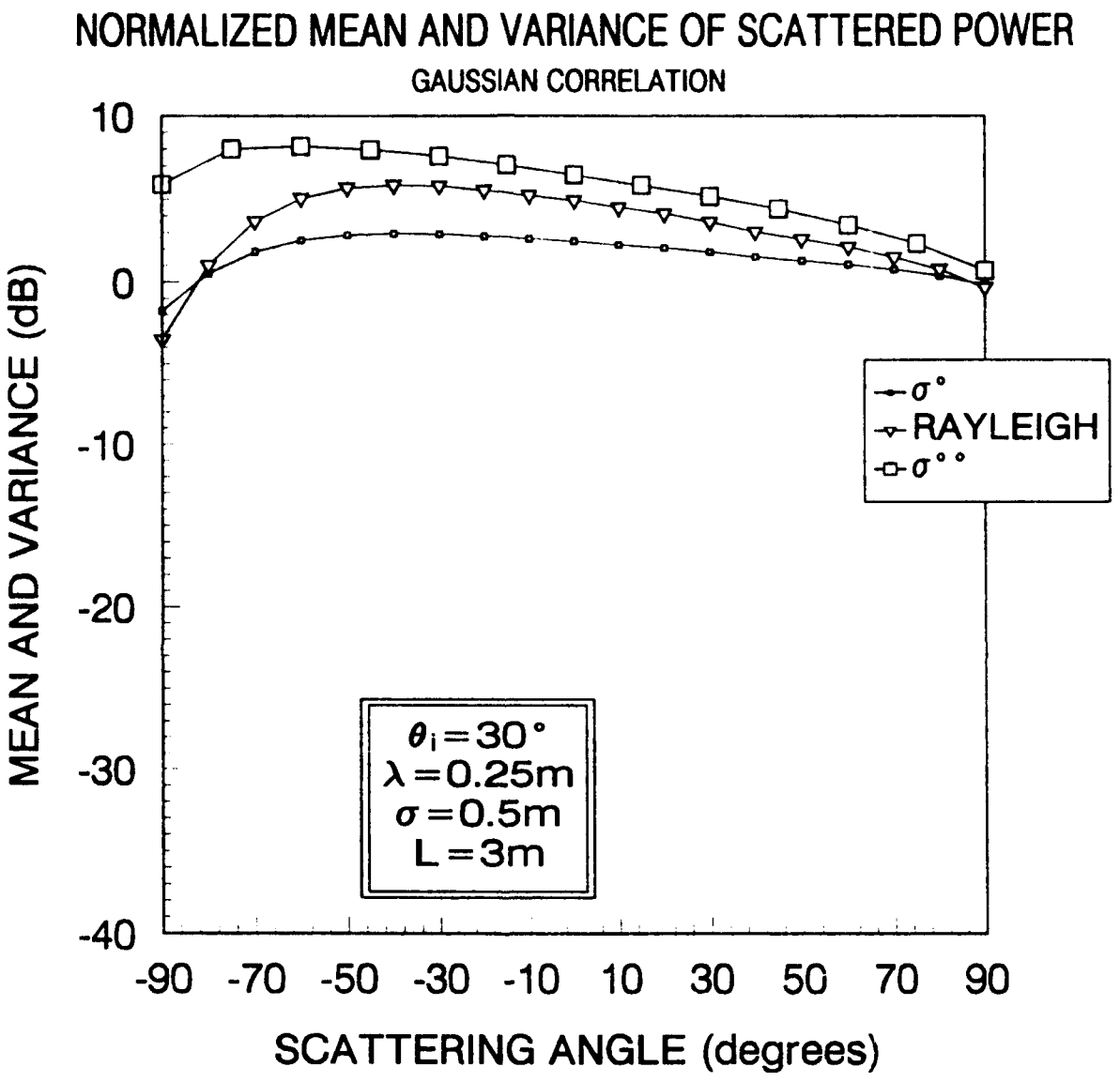


Figure 14. Normalized Mean and Variance for Gaussian Surface, $\theta_i = 30°$, $\lambda = 0.25m$, $\sigma = 0.5m$, $L = 3m$

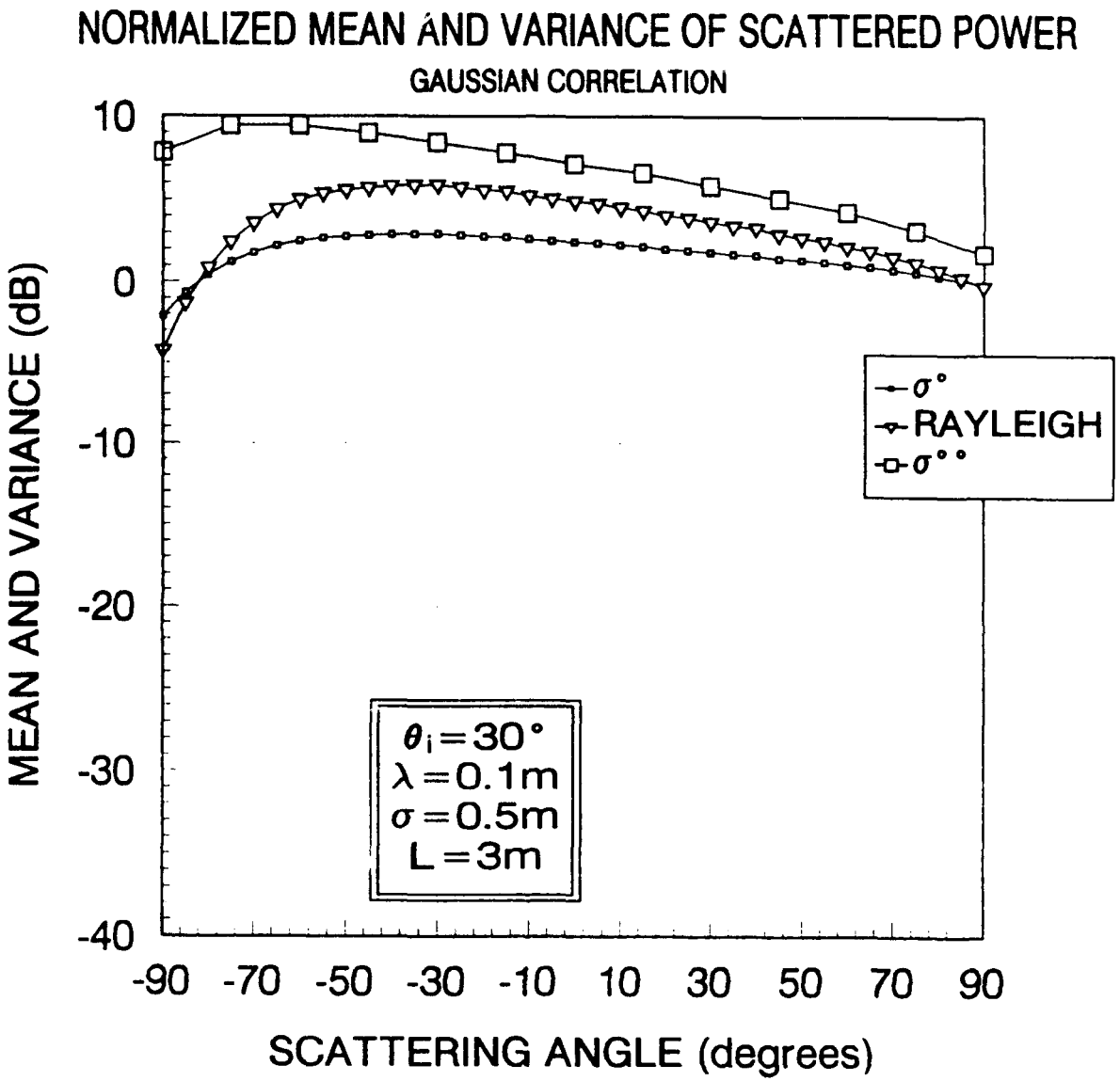


Figure 15. Normalized Mean and Variance for Gaussian Surface, $\theta_i = 30°$, $\lambda = 0.1$m, $\sigma = 0.5$m, $L = 3$m

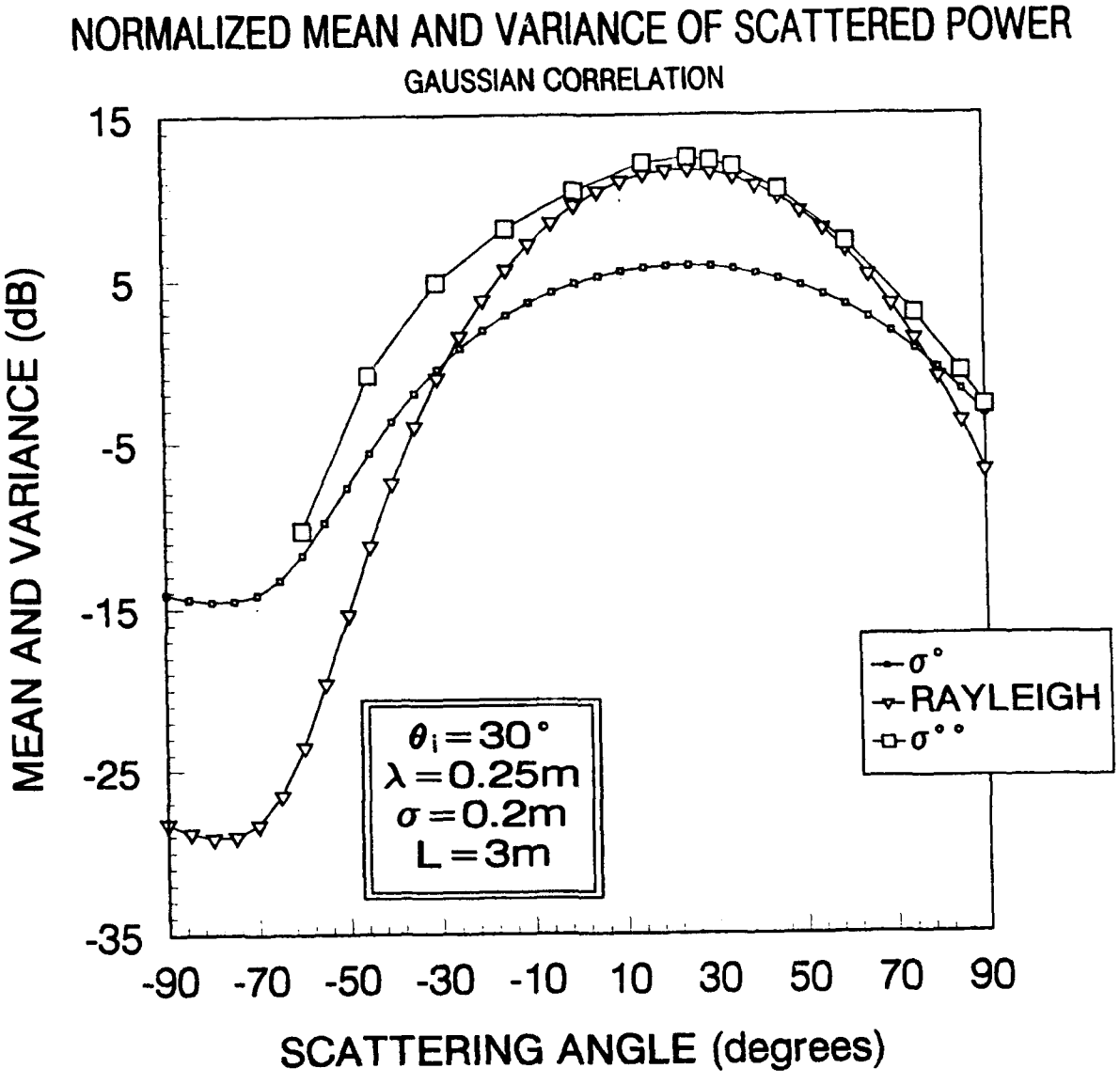


Figure 16. Normalized Mean and Variance for Gaussian Surface, $\theta_i = 30°$, $\lambda = 0.25$m, $\sigma = 0.2$m, $L = 3$m

The scattering from surfaces having a power law correlation function are shown in Figures 17 through 24. The results for the power law correlated surfaces are quite similar to those obtained with a Gaussian correlated surface. The trends due to changing wavelength, rms height, cell size, and scattering angles are all the same for surfaces with a power law correlation function as they were for the Gaussian correlated surface. One notable difference can be seen in comparing the results for the two surfaces. It is clear that in the backscatter region, where scattering from both surfaces deviates from Rayleigh, the deviation for the power law case is not as severe. This is a consequence of the increased number of scattering facets oriented to cause backscatter for the surface with a power law correlation. In comparing Figures 3 and 6 to Figures 2 and 5, it is obvious that the power law correlated surface is somewhat more jagged and will therefore have more scattering facets than the Gaussian correlated surfaces. The increase in the number of scattering facets results in scattering statistics that are closer to Rayleigh.

5. EXPONENTIAL CORRELATION FUNCTION RESULTS

The surfaces described by an exponential correlation function, as stated previously, are very jagged and have many vertical facets. Also, this type of surface has more fine scale structure than the other surfaces. Because of this, the scattering from this type of surface is quite different than that of the Gaussian or power law correlated surfaces. The most significant result found for the exponentially correlated surface is that the cell size has very little effect on the Rayleighness of the scattering statistics. The results for the exponentially correlated surface are shown in Figures 25 through 32. For cell sizes of L = 3m, no noticeable deviation from Rayleigh scattering occurs over the entire scattering region. Because at L = 3m the agreement with Rayleigh scattering was so close, larger cell sizes were not examined. The smallest cell size studied is L = 2m. In this case we see that there is still very little deviation from Rayleigh scattering. This is a very significant result because it means that for very small clutter cells, Rayleigh scattering can be assumed even for monostatic radars when the surface is described by an exponential correlation function. In the Gaussian and power law cases, the deviation from Rayleigh scattering is quite large in the backscatter region.

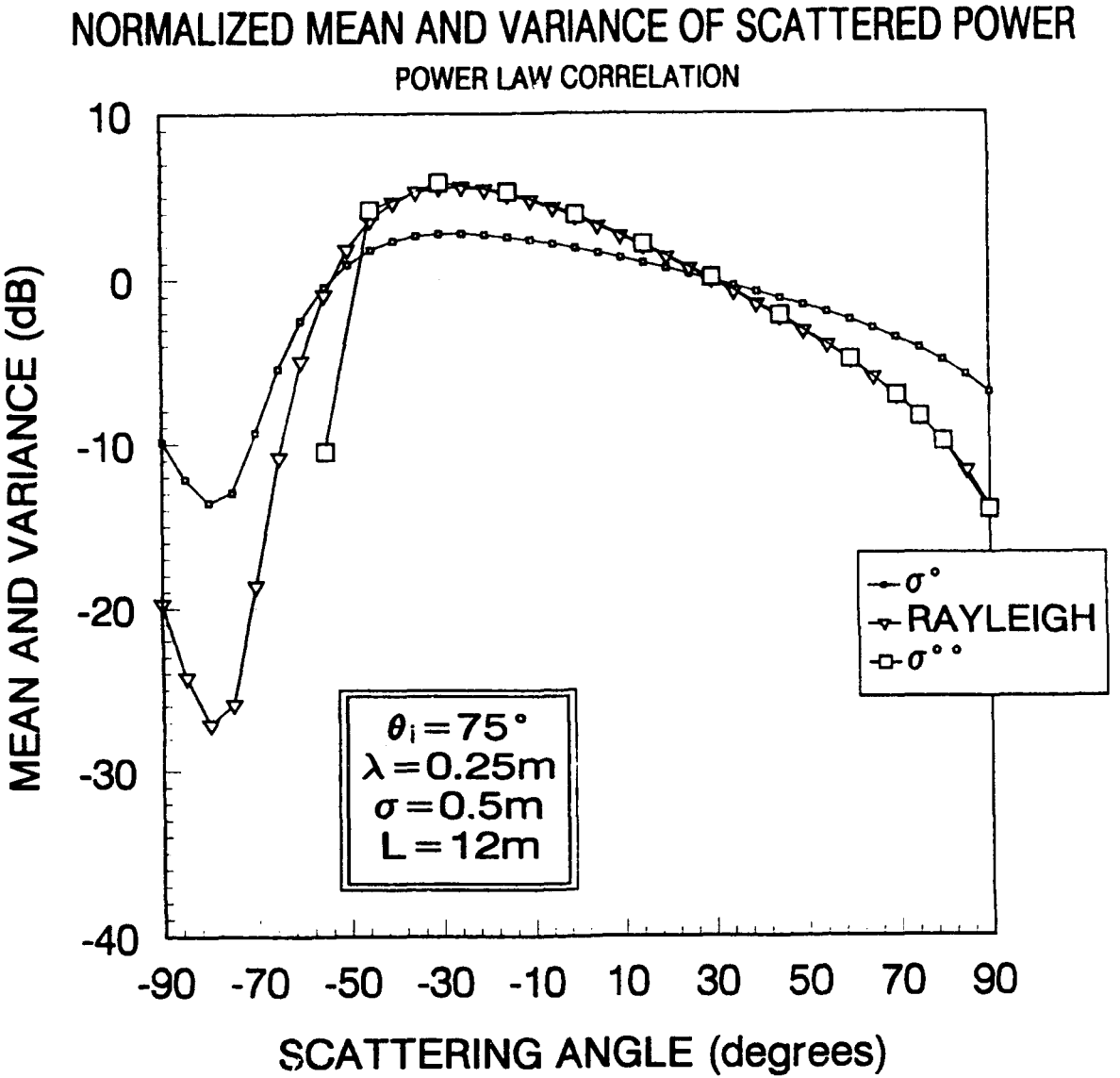


Figure 17. Normalized Mean and Variance for Power Law Surface, $\theta_i = 75°$, $\lambda = 0.25$m, $\sigma = 0.5$m, L = 12m

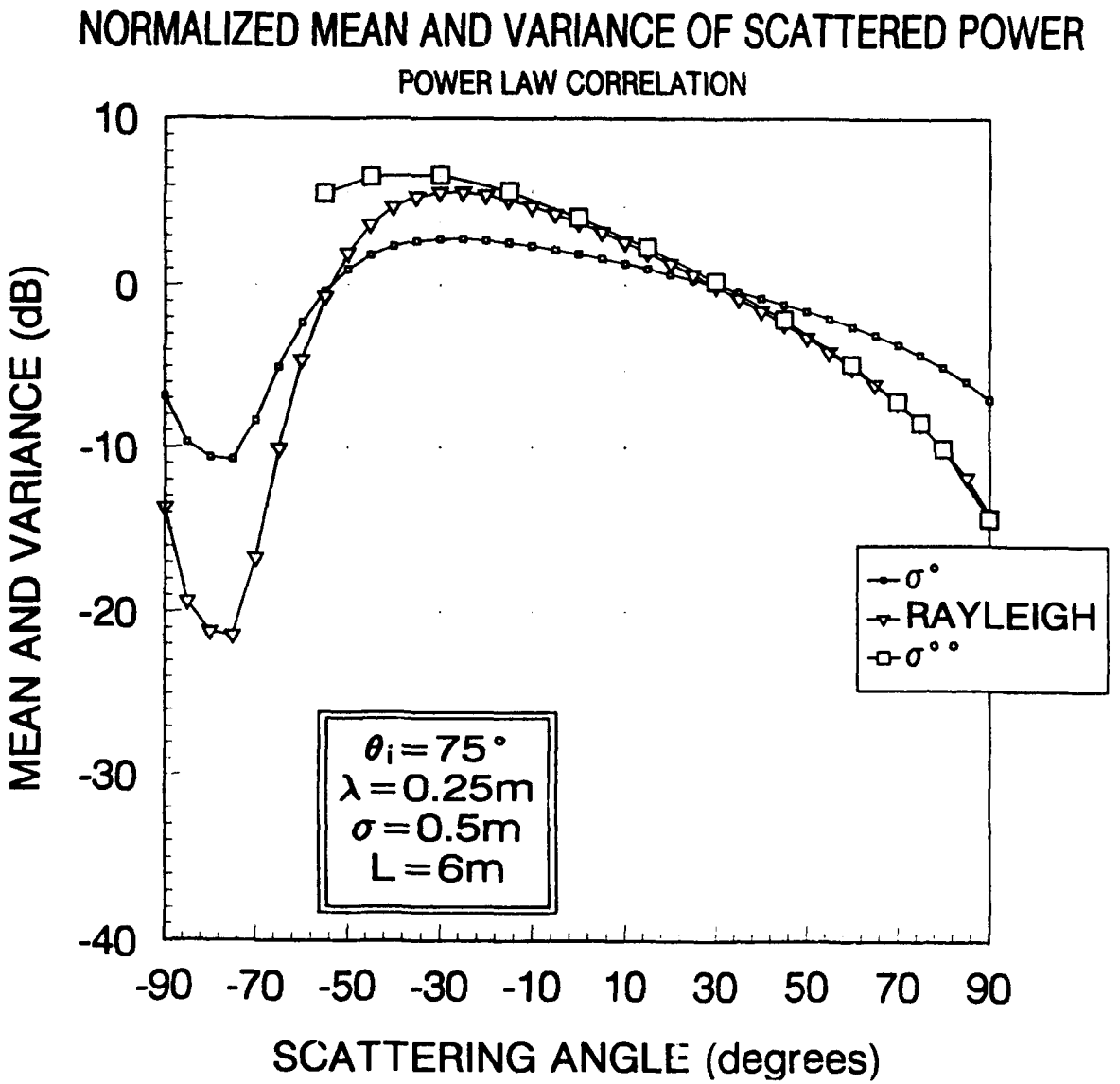


Figure 18. Normalized Mean and Variance for Power Law Surface, $\theta_i = 75°$, $\lambda = 0.25m$, $\sigma = 0.5m$, $L = 6m$

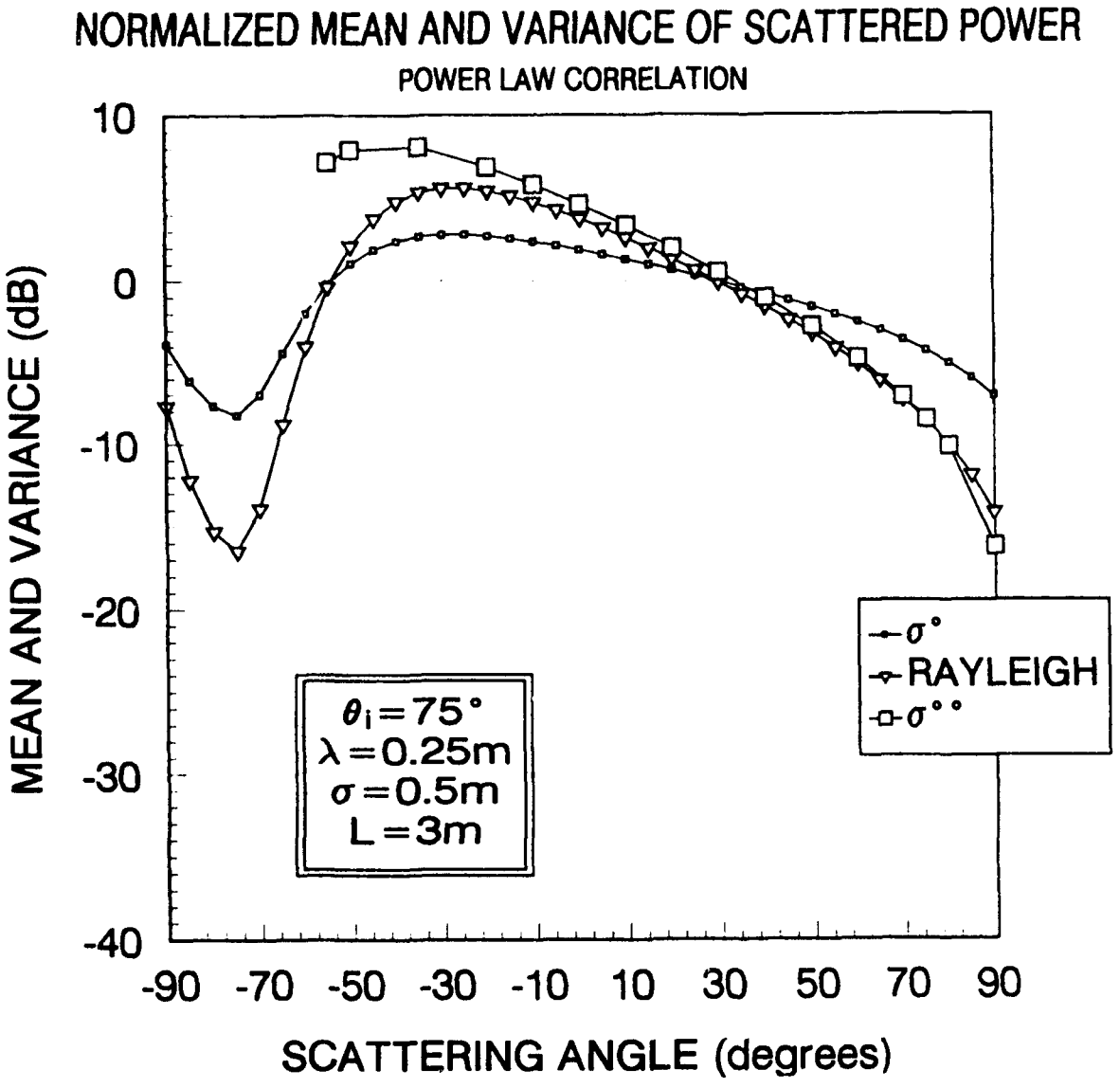


Figure 19. Normalized Mean and Variance for Power Law Surface, $\theta_i = 75°$, $\lambda = 0.25$m, $\sigma = 0.5$m, L = 3m

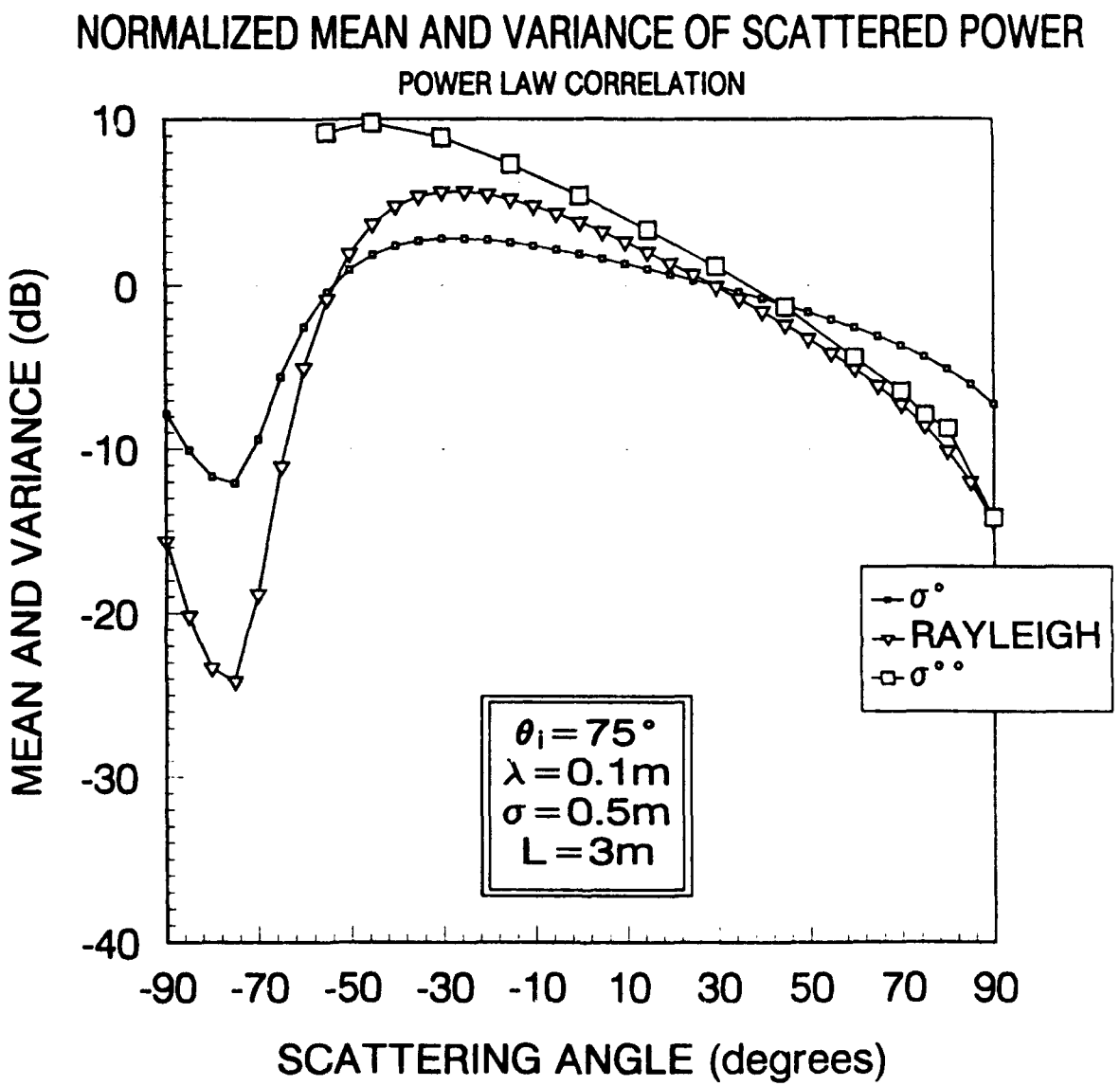


Figure 20. Normalized Mean and Variance for Power Law Surface. θ_i = 75°, λ = 0.1m, σ = 0.5m, L = 3m

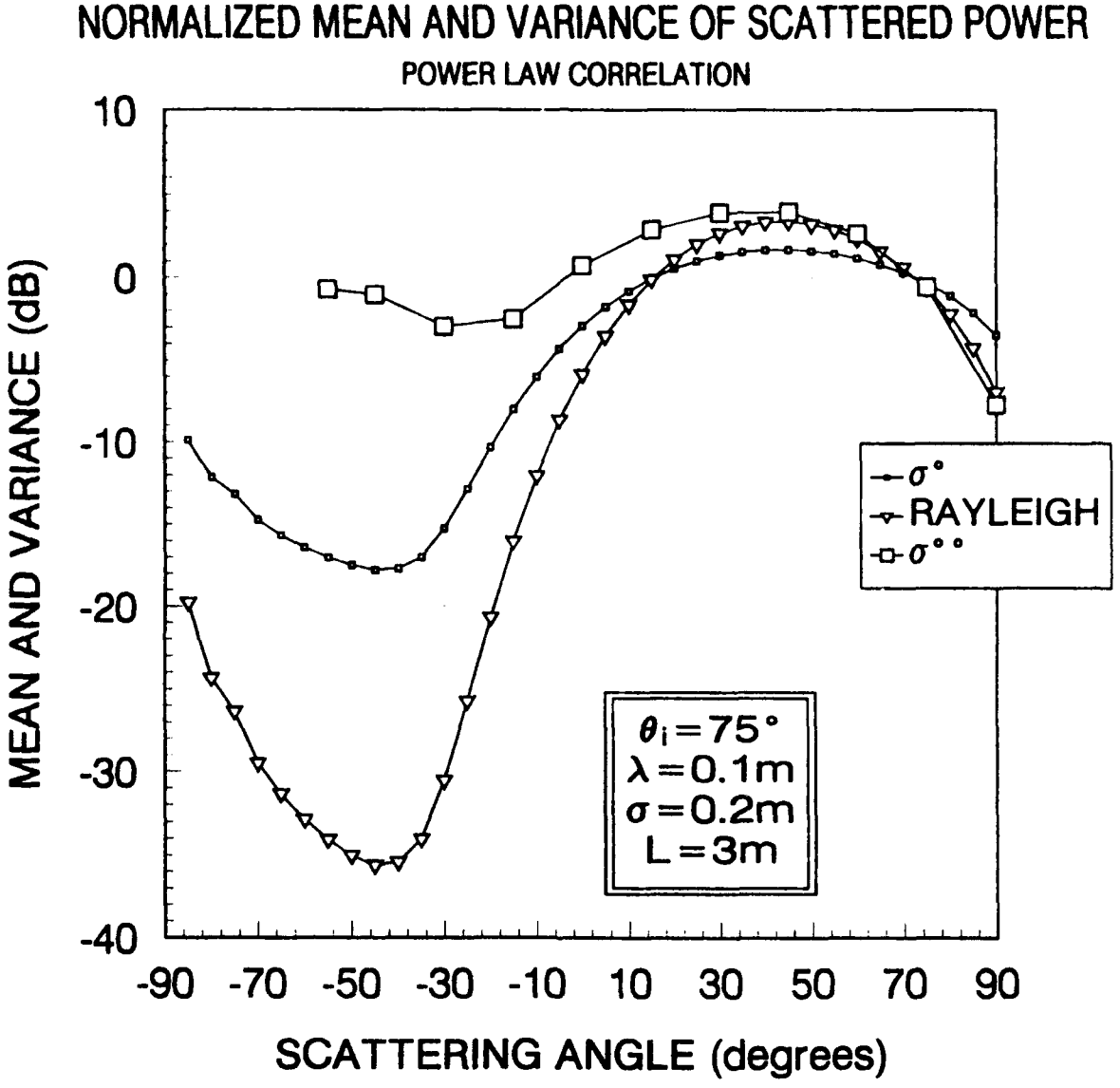


Figure 21. Normalized Mean and Variance for Power Law Surface, $\theta_i = 75°$, $\lambda = 0.1$m, $\sigma = 0.2$m, $L = 3$m

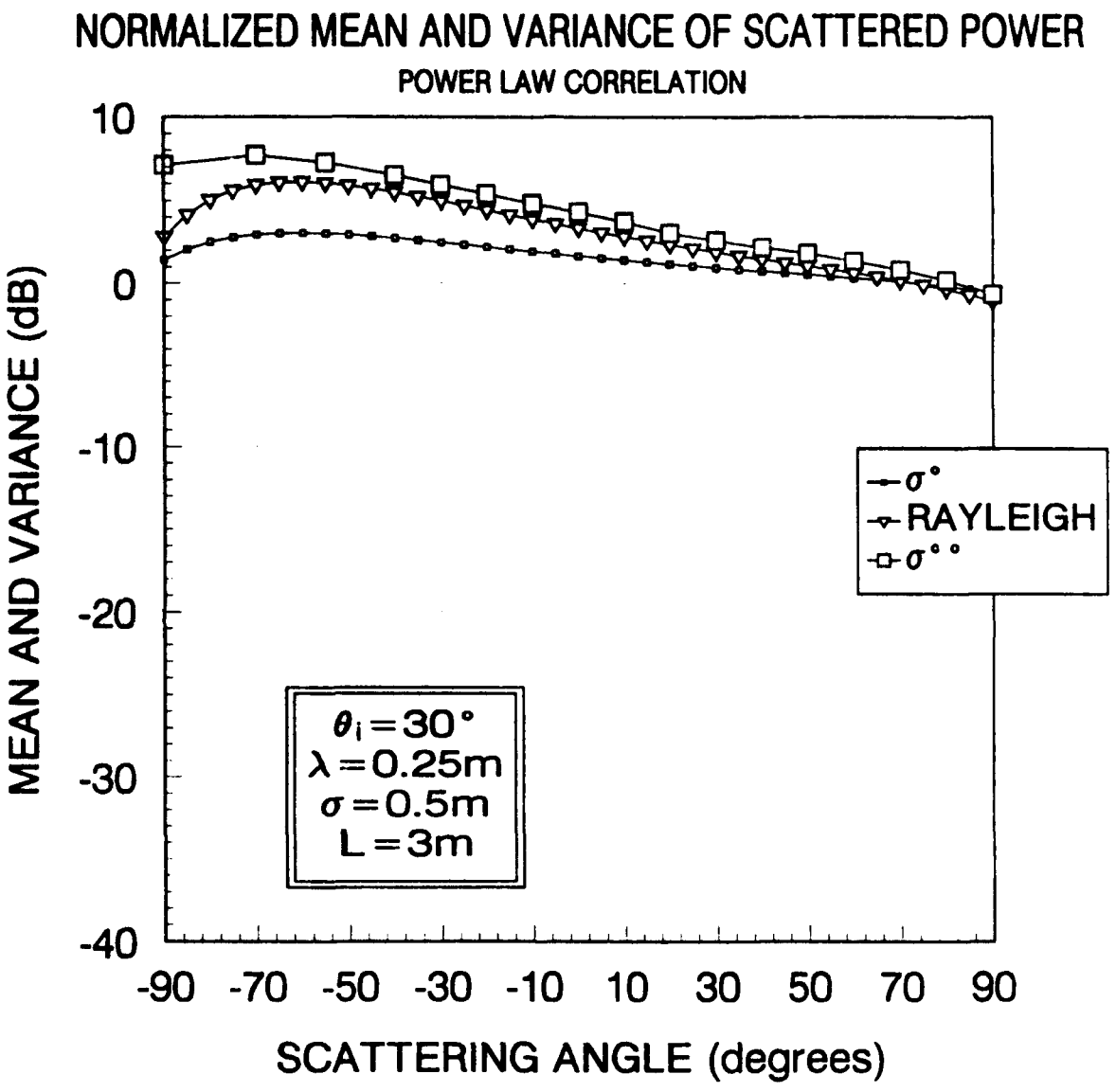


Figure 22. Normalized Mean and Variance for Power Law Surface, θ_i = 30°, λ = 0.25m, σ = 0.5m, L = 3m

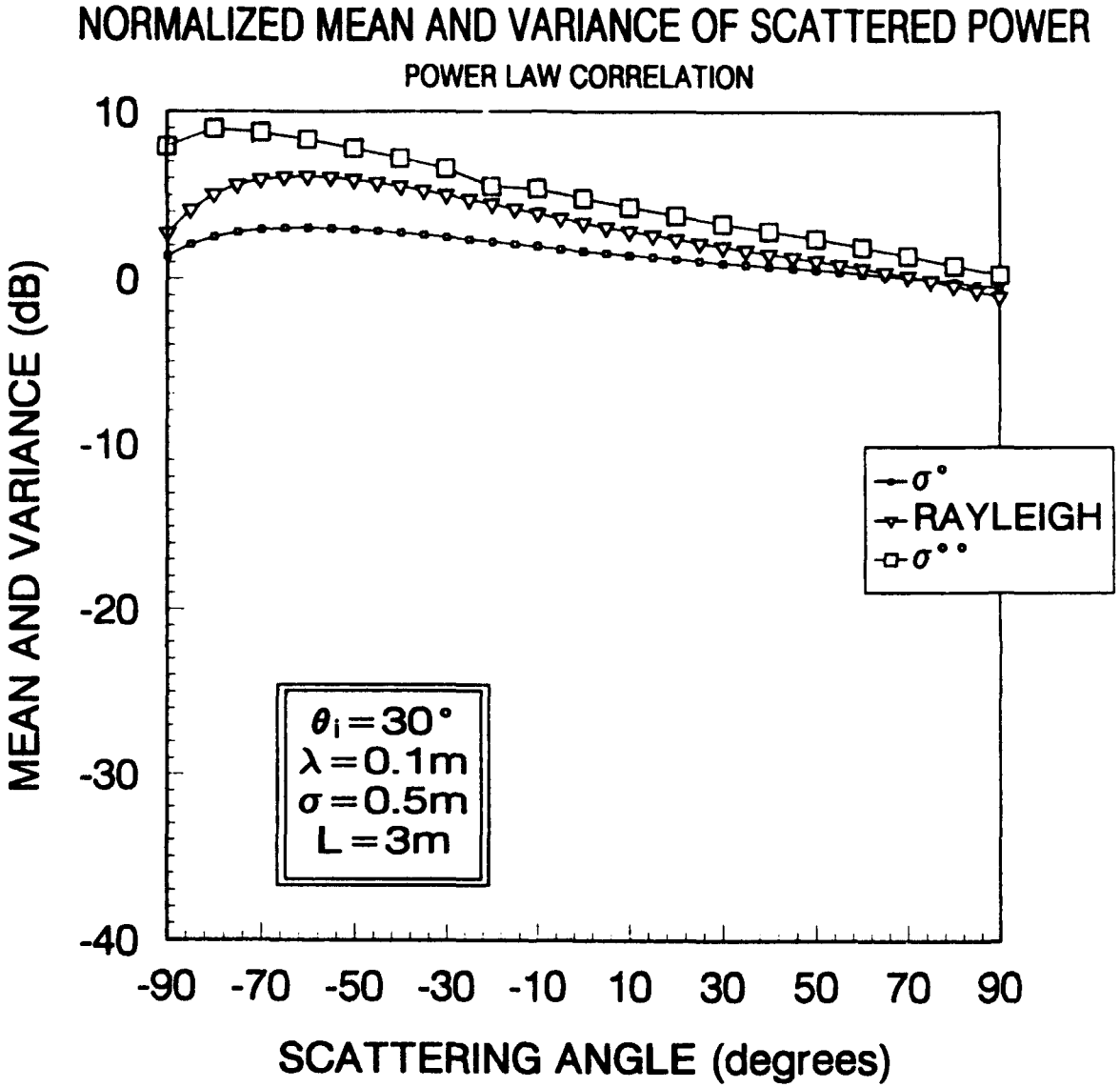


Figure 23. Normalized Mean and Variance for Power Law Surface, $\theta_i = 30°$, $\lambda = 0.1$m, $\sigma = 0.5$m, $L = 3$m

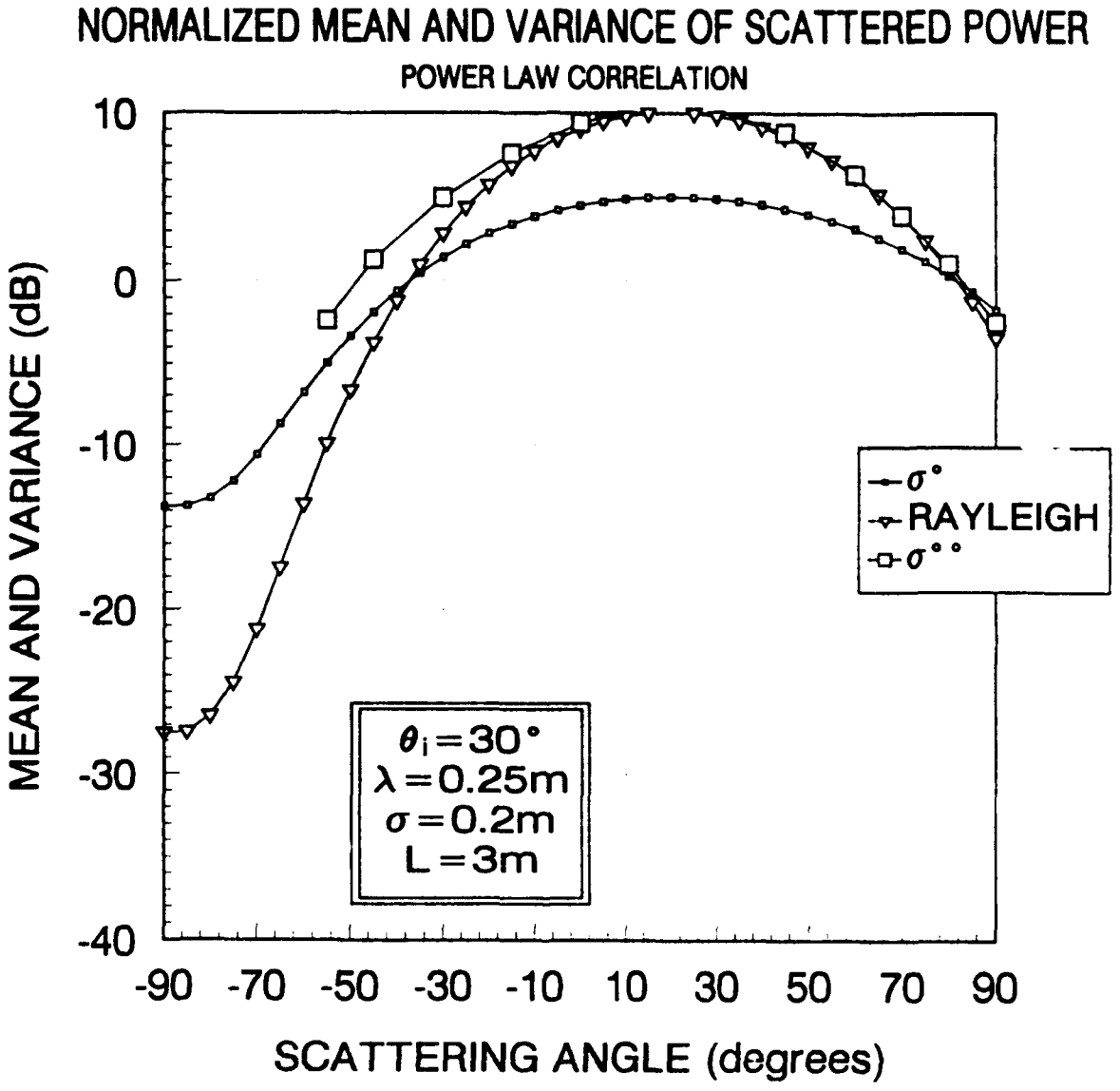


Figure 24. Normalized Mean and Variance for Power Law Surface, $\theta_i = 30°$, $\lambda = 0.25$m, $\sigma = 0.2$m, L = 3m

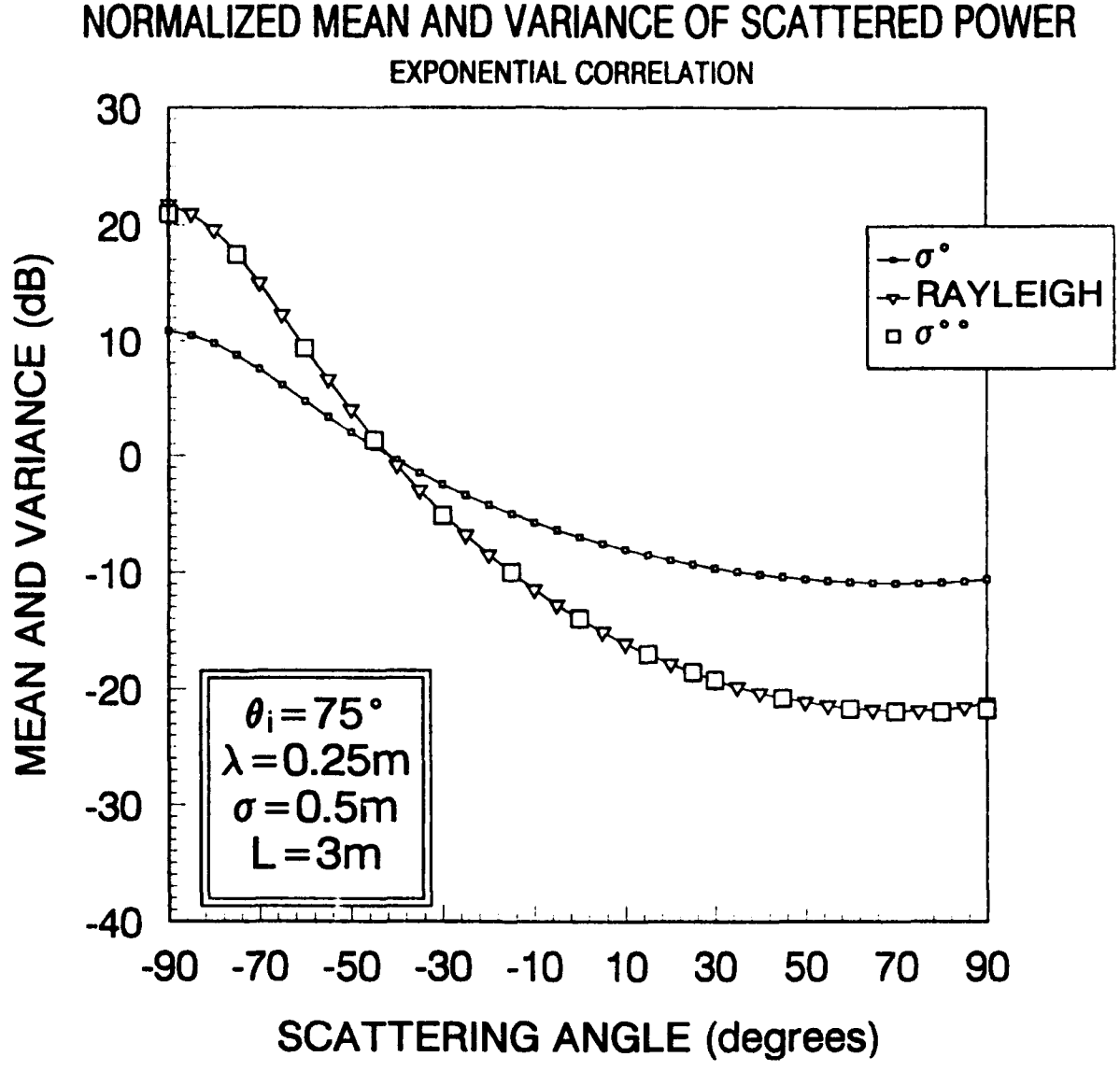


Figure 25. Normalized Mean and Variance for Exponential Surface, $\theta_i = 75°$, $\lambda = 0.25m$, $\sigma = 0.5m$, $L = 3m$

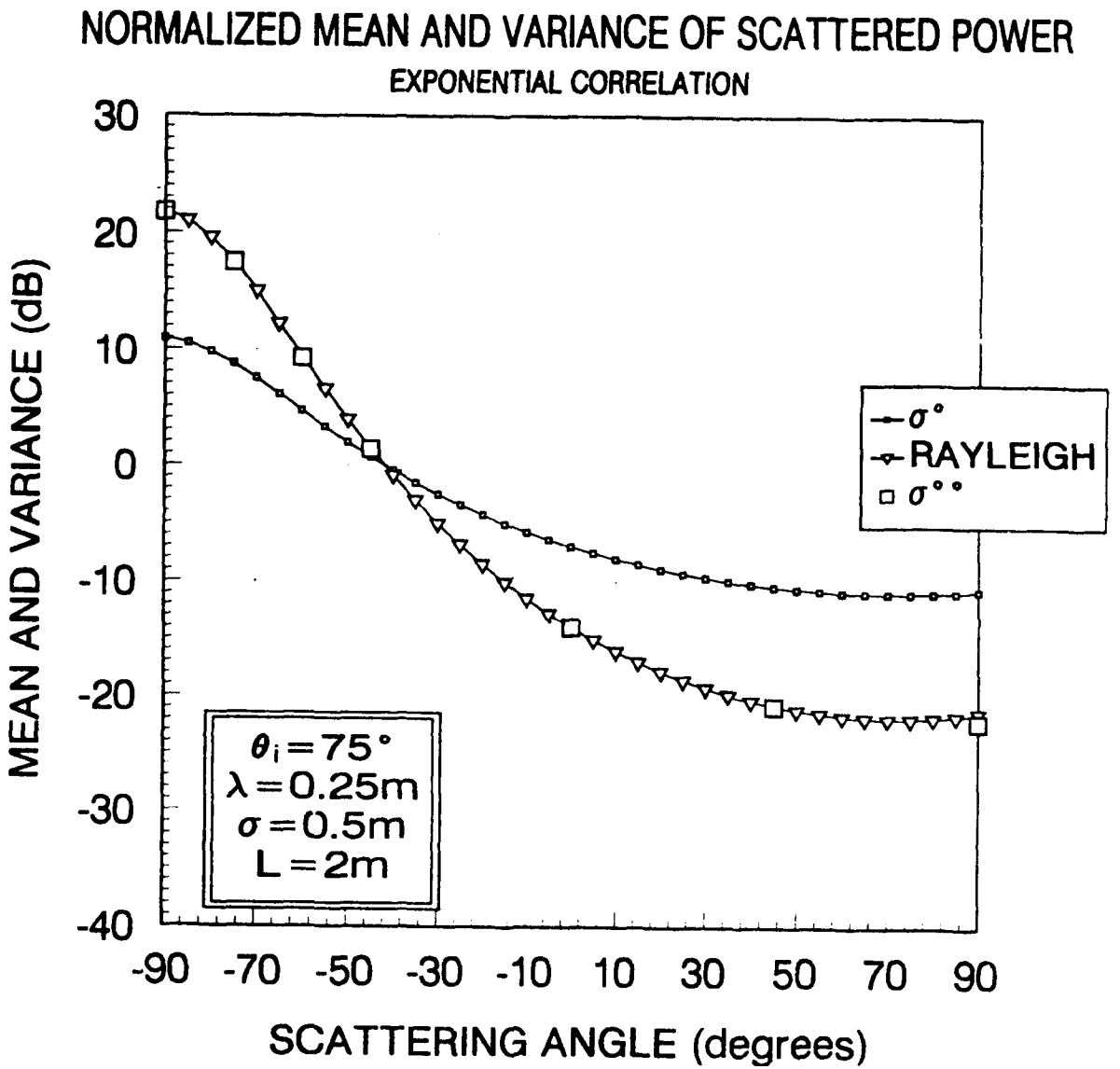


Figure 26. Normalized Mean and Variance for Exponential Surface, $\theta_i = 75°$, $\lambda = 0.25$m, $\sigma = 0.5$m, $L = 2$m

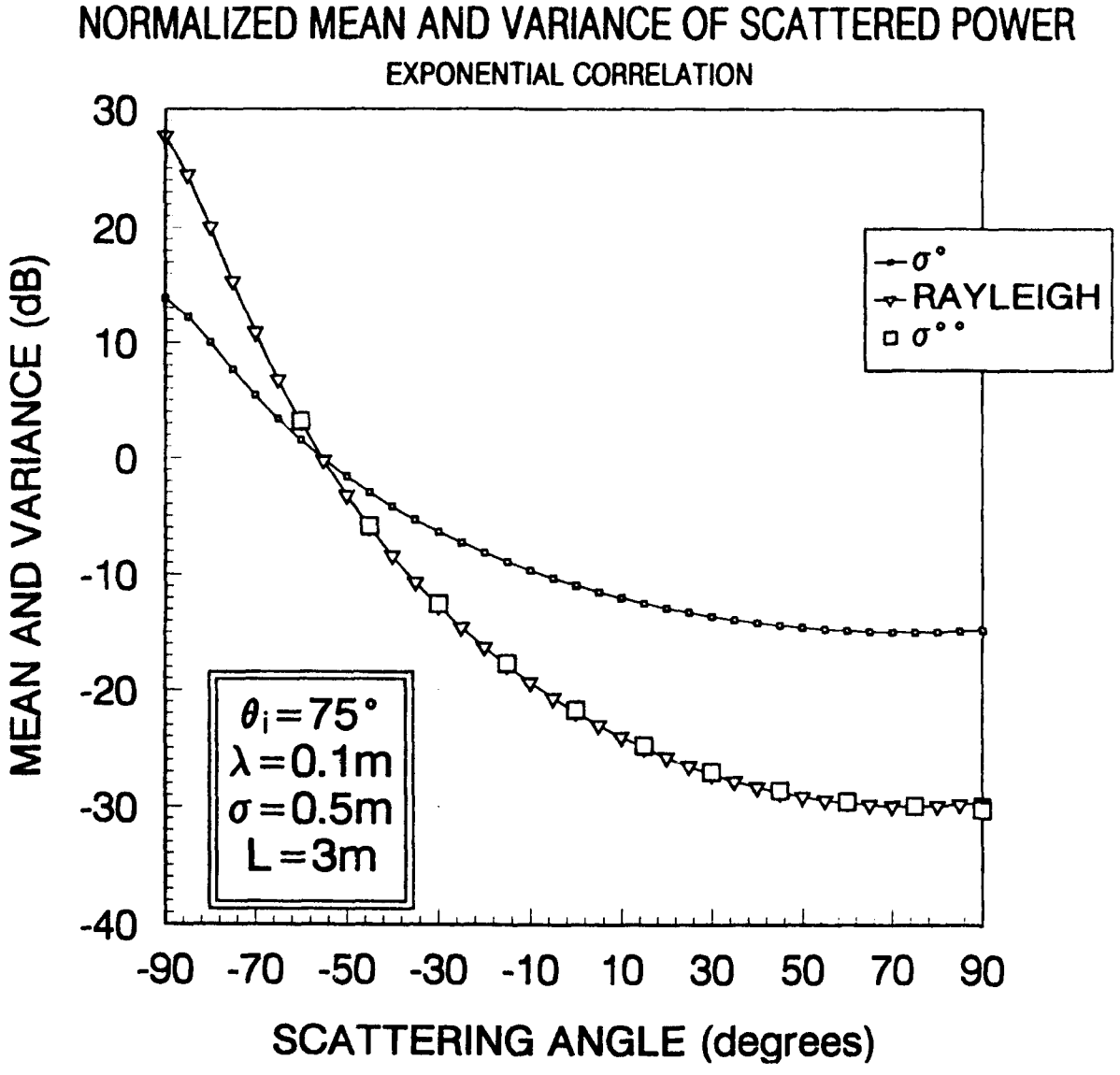


Figure 27. Normalized Mean and Variance for Exponential Surface, $\theta_i = 75°$, $\lambda = 0.1$m, $\sigma = 0.5$m, $L = 3$m

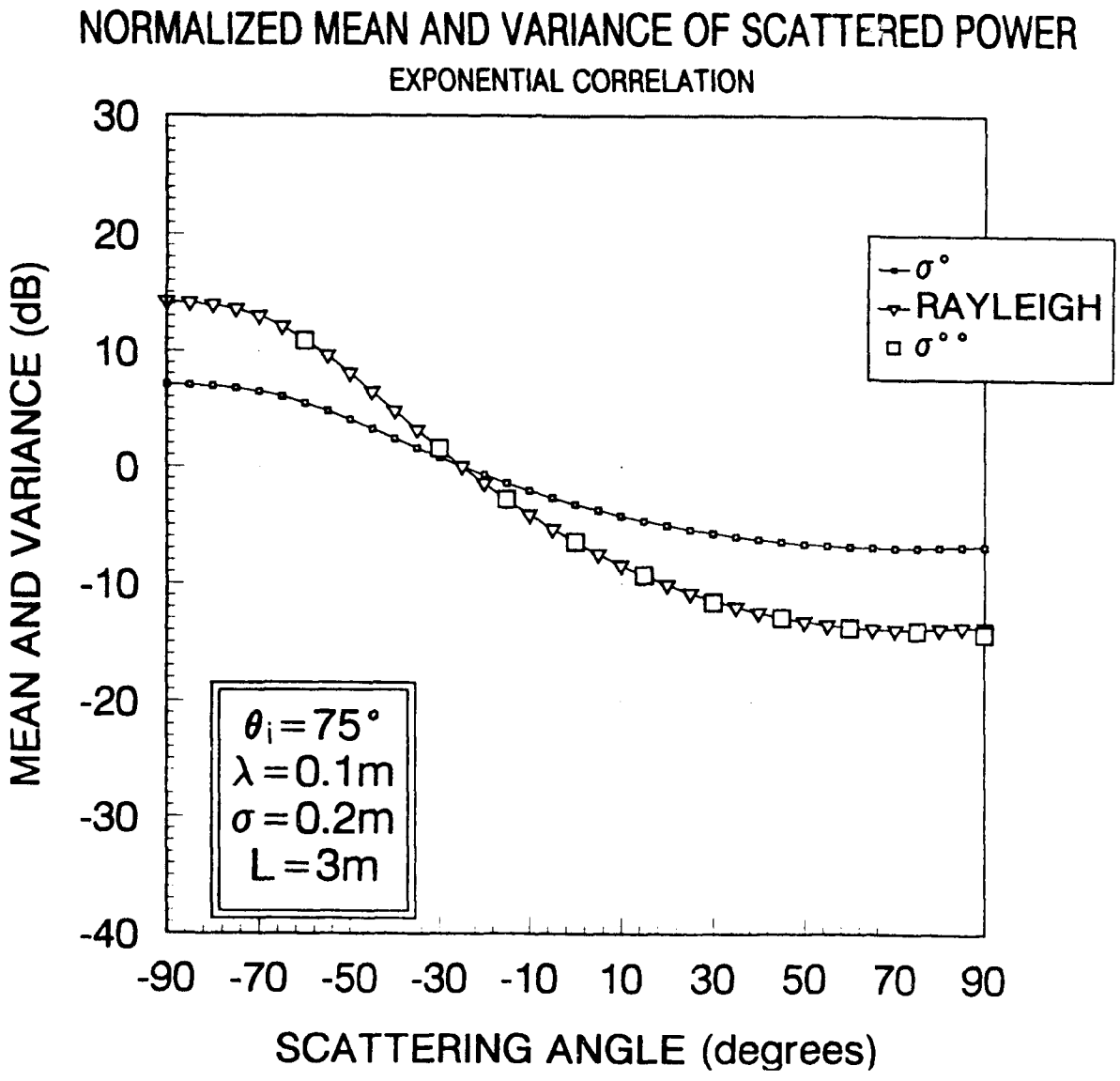


Figure 28. Normalized Mean and Variance for Exponential Surface, $\theta_i = 75°$, $\lambda = 0.1$m, $\sigma = 0.2$m, L = 3m

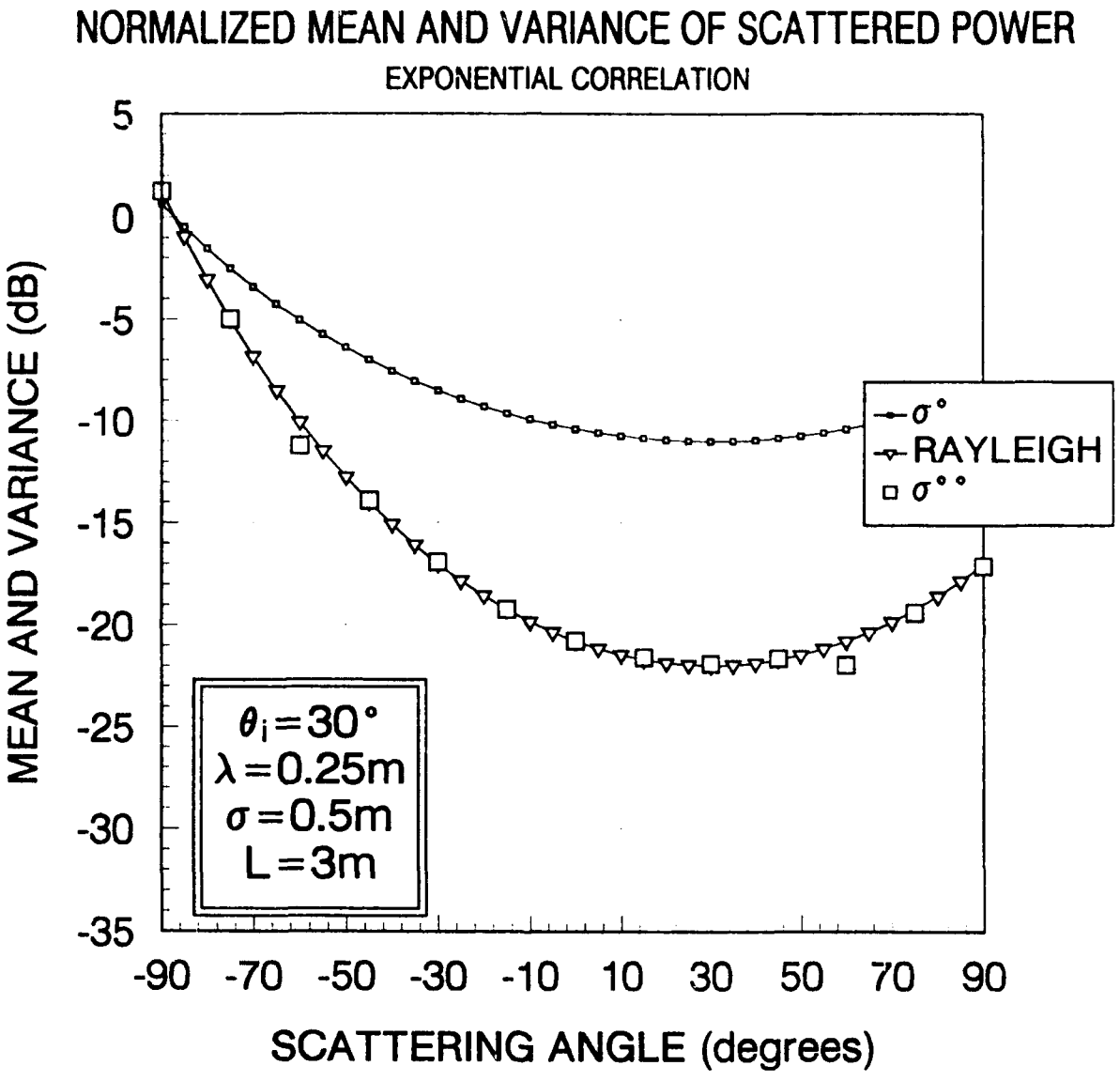


Figure 29. Normalized Mean and Variance for Exponential Surface, $\theta_i = 30°$, $\lambda = 0.25m$, $\sigma = 0.5m$, $L = 3m$

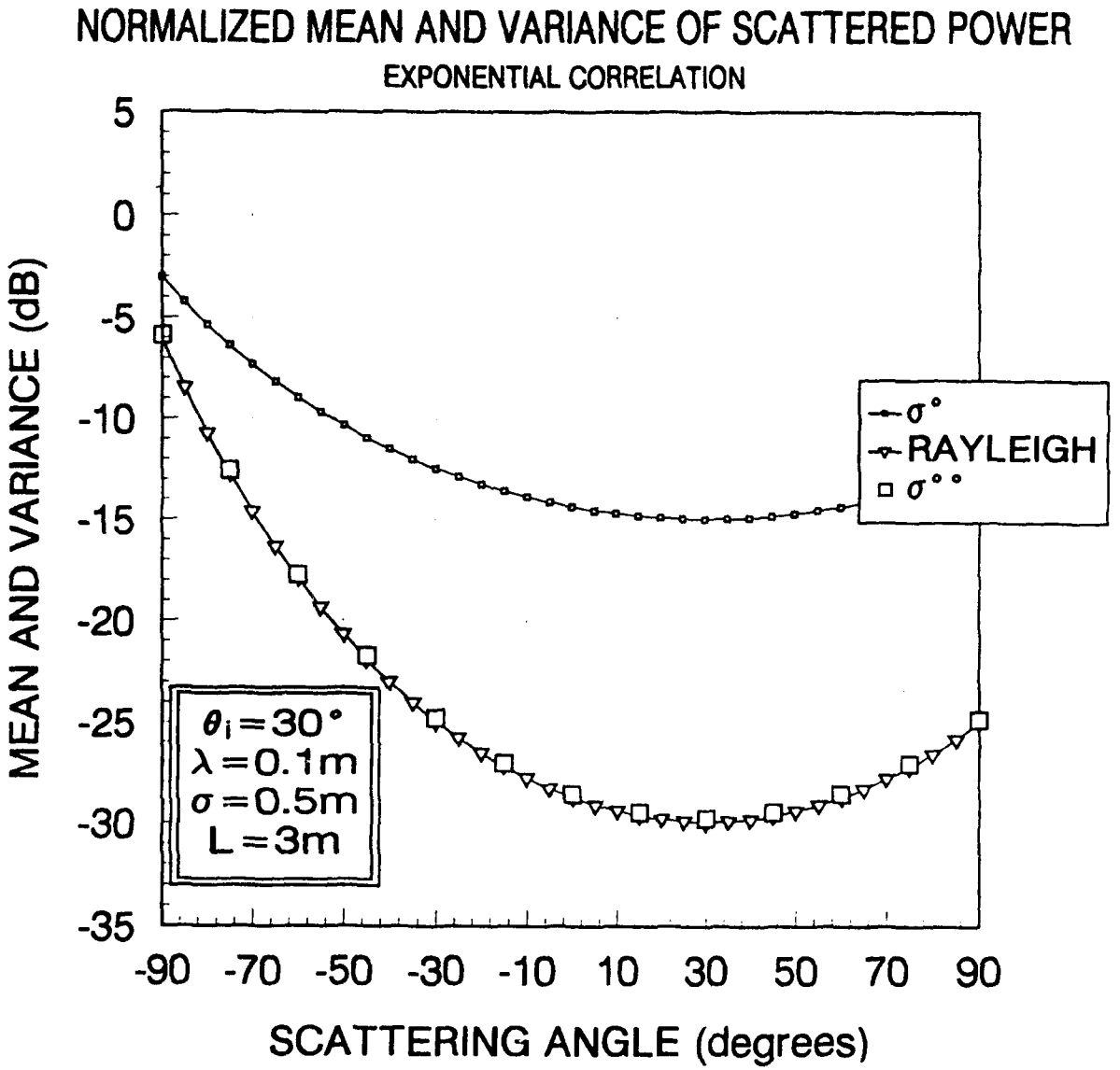


Figure 30. Normalized Mean and Variance for Exponential Surface, $\theta_i = 30°$, $\lambda = 0.1$m, $\sigma = 0.5$m, $L = 3$m

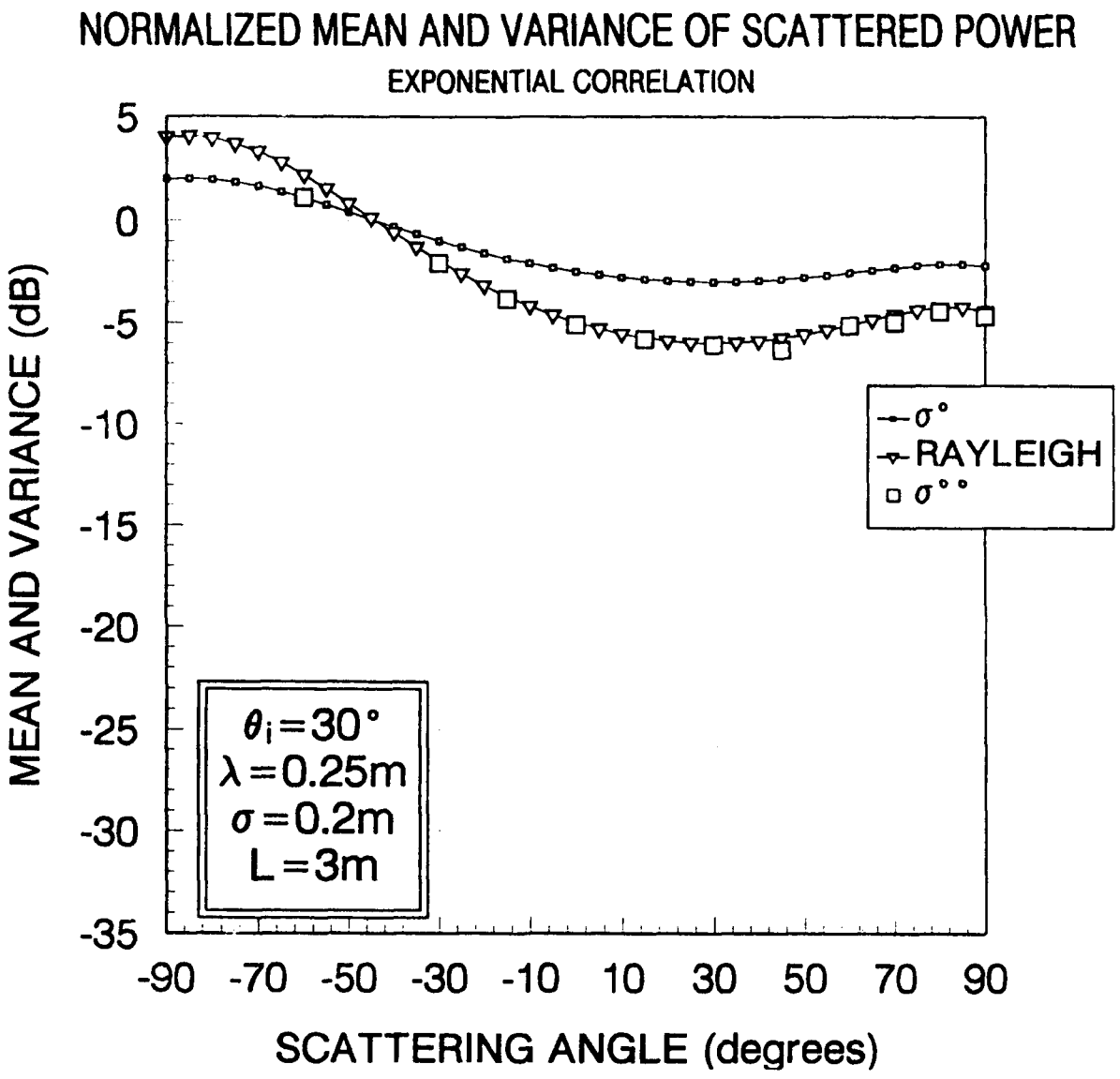


Figure 31. Normalized Mean and Variance for Exponential Surface, $\theta_i = 30°$, $\lambda = 0.25$m, $\sigma = 0.2$m, $L = 3$m

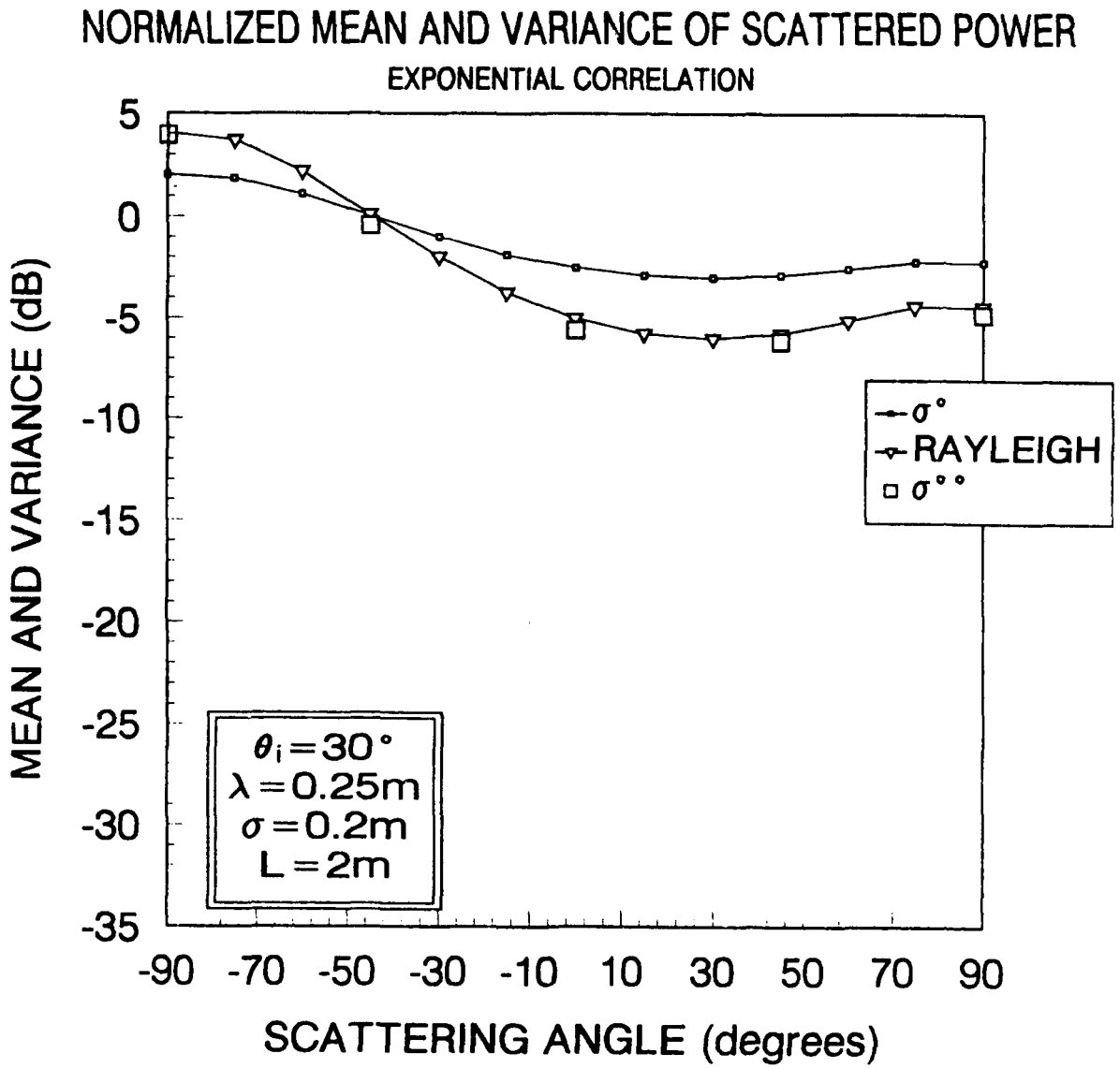


Figure 32. Normalized Mean and Variance for Exponential Surface, $\theta_i = 30°$, $\lambda = 0.25$m, $\sigma = 0.2$m, $L = 2$m

The reason why the exponentially correlated surface exhibits Rayleigh scattering for small cell sizes while the other surfaces do not can be understood by considering the geometry of the exponentially correlated surface shown in Figures 4 and 7. In the Gaussian and power law cases, where the surface is smoothly varying, the deviation from Rayleigh could be quite significant for small cell sizes, particularly in the backscatter region, because there are so few scattering facets oriented properly to cause backscatter. However, the exponential surface, which is very jagged and has many vertical facets, has a sufficient number of scattering surfaces oriented in all directions to cause the scattering to remain Rayleigh over the entire region of scattering angles. For the exponential surface, the direction in which the maximum scattering occurs is -90°, due to the large number of vertical facets, while the minimum scattering is in the specular direction.

The mean value of the scattering from an exponentially correlated surface is almost always lower than σ^0 for the Gaussian correlated surface. For the 30° angle of incidence, when the wave is incident upon vertical facets, a considerable amount of the energy will be reflected downward, and since second reflections are not accounted for in this study, this energy disappears. However, for the 75° angle of incidence, much of the scattering is reflected back by the vertical facets causing very high backscatter and very little forward scatter. In comparing the mean value of the scattering from the Gaussian correlated surface and the exponential surface we see that σ^0 is higher for the exponential correlation in the backscatter region but lower than scattering from Gaussian correlated surfaces in the forward scatter region. This result agrees with intuition because higher backscatter would be expected from urban areas for low grazing angles. Table 1 compares σ^0 for the backscatter direction for the exponential and Gaussian correlated surfaces.

Table 1. Comparison of Backscatter for Exponential and Gaussian Correlated Surfaces.

θ_i^0	λ (m)	σ (m)	L (m)	σ^0_{exp} (dB)	σ^0_{gauss} (dB)
30	0.25	0.5	3	-8.50	2.90
30	0.1	0.5	3	-12.48	2.95
75	0.25	0.5	3	8.79	-9.18
75	0.1	0.2	3	6.81	-13.65

6. CONCLUSIONS

In this report it is shown that as the size of a clutter cell is decreased, the distribution of the power scattered from that clutter cell depends on several factors. The wavelength of the incident field, the roughness of the surface, and the incident angle will all have some effect on the Rayleighness of the scattered power, as discussed in the report by Sharpe[2], but the results

shown in this report demonstrate that the correlation function of the surface can have a dramatic influence on the statistics of the scattered power. For surfaces described by a Gaussian correlation function, deviation from Rayleigh scattering can be significant, particularly in the backscatter region. For surfaces having a power law correlation function, the scattering also shows a noticeable departure from Rayleigh in the backscatter region, although the difference is not quite as large as for the Gaussian correlated surfaces.

For surfaces having an exponential correlation function, the scattering statistics did not deviate from Rayleigh over the entire scattering region studied, even for very small sizes. This is not surprising because there is still a large number of scattering facets in the exponentially distributed surface, even for the very small cell sizes, which causes the scattering to remain Rayleigh, while the number of scattering facets on the Gaussian distributed surface decreases rapidly with cell size. From these results we can conclude that the number of scattering facets in a clutter cell has a significant effect on the statistics of the scattered power.

Knowledge of the type of terrain which is to be encountered in a radar system can therefore be very important. If the expected clutter is known to have a Gaussian correlation or power law correlation, problems may be encountered due to large returns from the clutter. However, if the surface causing the clutter is known to have an exponential correlation function, the assumption of Rayleigh scattering will be valid, even for very small cell sizes. Because an exponential correlation function is used to describe cities and other urban areas, this is an important result.

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1. Papa, R.J. and Woodworth, M.B. (1991) *The Mean and Variance of Diffuse Scattered Power as a Function of Clutter Resolution Cell Size*, RADC-TR-91-09.

2. Sharpe, L.M. (1991) *Analytical Characterization of Bistatic Scattering from Gaussian Distributed Surfaces*, RL-TR-91-351.

3. Ruck, G.T., Barrick, D.E., Stuart, W.D., and Krichbaum, C.K. (1970) *Radar Cross Section Handbook*, 2. Plenum Press, New York.

4. Cohen, E., Bell, M., Lin, C., Holst, J., and Tomasanis, D. (1988) *Mathematical Analysis and Programming for Electromagnetic Applications*, RADC-TR-88-18. ADB124496.

5. Barrick, D.E., and Peake, W.H. (1967) *Scattering from Surfaces with Different Roughness Scales: Analysis and Interpretation*, BAT-197A-10-3.

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A1. Geist, J.M. (1979) Computer generation of correlated Gaussian random variables, *Proc. IEEE*, **67** (No. 5).

A2. Stark, H. and Woods, J.W. (1979) *Probability, Random Processes, and Estimation Theory for Engineers*. Prentice Hall, Inc., Englewood Cliffs, New Jersey.

A3. IMSL, Inc. (1987) Subroutine CHFAC and Subroutine RNMVN, *Fortran Subroutines for Mathematical Applications*, IMSL, Inc., Houston, Texas.

Appendix A
Generation of the Correlation Matrix and Correlated Surfaces

To generate a surface having a Gaussian height distribution function and a specified correlation function, $C(\tau)$, one must take the desired correlation matrix, transform it into an upper triangular matrix, and multiply it by the set of independent Gaussian distributed heights.[A1] Let $u = [u_1, u_2, \ldots u_N]^T$ be the set of uncorrelated, Gaussian distributed random variables, and C be the correlation matrix.

The correlation matrix is derived directly from the correlation function, $C(\tau)$. The deminsions of the matrix are equal to the number of points, N, on the surface of length, L. The distance between two adjacent points on the surface is $d=L/(N-1)$. τ is equal to the distance between point i and point j and is given by $\tau=d|i-j|$. The matrix can then be filled in by using the correlation function and making $C_{ij}=C(\tau)$. The matrix will be square and symmetric.

For example, if a surface is 10 units in length and we want to determine the correlation matrix of six points on the surface, we would have L=10, and N=6. Therefore, d=2 and $\tau=2|i-j|$. For an exponential correlation function, $C(\tau)=\exp(-|\tau|/T)$ where T is the correlation length which we will set to one for this example. We can then fill in the correlation matrix by the equation, $C_{ij}=\exp(-2|i-j|)$. For i=j, $C_{ij}=1$. Also, $C_{ij}=C_{ji}$.

[A1] Geist, J.M. (1979) Computer generation of correlated Gaussian random variables, *Proc. IEEE*, **67** (No. 5).

$|i-j| = |j-i| = 1$, $C_{ij} = \exp(-2) = 0.135$
$|i-j| = |j-i| = 2$, $C_{ij} = \exp(-4) = 0.018$
$|i-j| = |j-i| = 3$, $C_{ij} = \exp(-6) = 0.002$
$|i-j| = |j-i| = 4$, $C_{ij} = \exp(-8) = 0.0003$
$|i-j| = |j-i| = 5$, $C_{ij} = \exp(-10) \simeq 0$

The correlation matrix for the exponential correlation function is then:

$$C_{ij} = \begin{bmatrix} 1 & 0.135 & 0.018 & 0.002 & 0.0003 & 0 \\ 0.135 & 1 & 0.135 & 0.018 & 0.002 & 0.0003 \\ 0.018 & 0.135 & 1 & 0.135 & 0.018 & 0.002 \\ 0.002 & 0.018 & 0.135 & 1 & 0.135 & 0.018 \\ 0.0003 & 0.002 & 0.018 & 0.135 & 1 & 0.135 \\ 0 & 0.0003 & 0.002 & 0.018 & 0.135 & 1 \end{bmatrix}$$

The next step is to convert the matrix into upper triangular form. The method used to transform the correlation matrix into upper triangular form is the Cholesky decomposition.[A2] Let A be the upper triangular representation of the corelation matrix. Subroutine CHFAC, which is part of the IMSL library, performs the decomposition.[A3] Subroutine RNMVN, also from the IMSL library, generates psuedorandom Gaussian distributed numbers, u, to simulate heights having a Gaussian distribution function. These heights are then multiplied by the upper triangular matrix found in CHFAC, resulting in a vector, y

$$y = A\,x \qquad \text{(A2)}$$

The vector y represents the heights of a surface that is Gaussian distributed and has a correlation function, $C(\tau)$.

[A2] Stark, H. and Woods, J.W. (1979) *Probability, Random Processes, and Estimation Theory for Engineers*, Prentice Hall, Inc., Englewood Cliffs, New Jersey.

[A3] MSL, Inc. (1987) Subroutine CHFAC and Subroutine RNMVN, *Fortran Subroutines for Mathematical Applications*, IMSL, Inc., Houston, Texas.

Appendix B

Solving the Integral for Scattered Power Over σ

The mean value of the scattered power from a one dimensionally rough surface of length L is given by:

$$\sigma^0 = \left[\frac{\pi F^2}{2 L \lambda}\right] \int_{-L/2}^{L/2} dx_1 \int_{-L/2}^{L/2} dx_2 \, F(\tau) \tag{B1}$$

where:

$$\tau = x_1 - x_2$$

$$F(\tau) = \left[\chi_2 - \chi_1 \chi_1^*\right] \cos\left(v_x \tau\right)$$

Perform a change of variables:

$$\tau = x_1 - x_2 \qquad x_1 = (1/2)(\tau + \sigma)$$
$$\sigma = x_1 + x_2 \qquad x_2 = (1/2)(\sigma - \tau) \tag{B2}$$

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial \tau} & \frac{\partial x_1}{\partial \sigma} \\ \frac{\partial x_2}{\partial \tau} & \frac{\partial x_2}{\partial \sigma} \end{vmatrix} = \frac{1}{2} .$$

Determine new limits:

$$\tau_{max} = x_{1,max} - x_{2,min} = \frac{L}{2} - \left(-\frac{L}{2}\right) = L$$
$$\tau_{min} = x_{1,min} - x_{2,max} = -\frac{L}{2} - \frac{L}{2} = -L$$
$$\sigma_{max} = x_{1,max} + x_{2,max} = \frac{L}{2} + \frac{L}{2} = L \tag{B3}$$
$$\sigma_{min} = x_{1,min} + x_{2,min} = -\frac{L}{2} + \left(-\frac{L}{2}\right) = -L$$

The integral is now over $F(\tau)$ as shown in Figure B1.

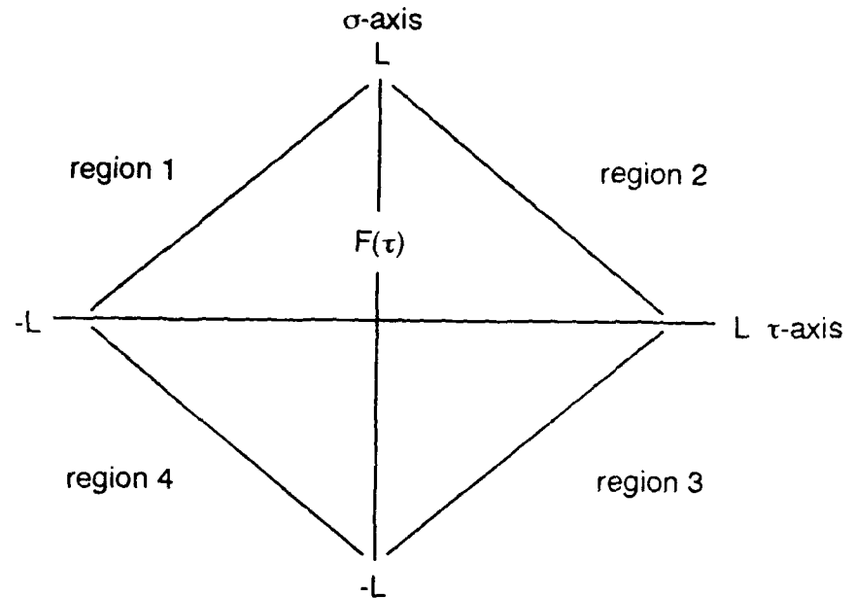


Figure B1. New Integration Area

$$\text{Region 1:} \int_{-L}^{0} d\tau\, F(\tau) \int_{0}^{L+\tau} d\sigma = \int_{-L}^{0} (L + \tau)\, F(\tau)\, d\tau$$

$$\text{Region 2:} \int_{0}^{L} d\tau\, F(\tau) \int_{0}^{L-\tau} d\sigma = \int_{0}^{L} (L - \tau)\, F(\tau)\, d\tau$$

$$\text{Region 3:} \int_{0}^{L} d\tau\, F(\tau) \int_{-L+\tau}^{0} d\sigma = \int_{0}^{L} (L - \tau)\, F(\tau)\, d\tau$$

$$\text{Region 4:} \int_{-L}^{0} d\tau\, F(\tau) \int_{-L-\tau}^{0} d\sigma = \int_{-L}^{0} (L - \tau)\, F(\tau)\, d\tau$$

Combining regions 1 and 4 and regions 2 and 3 and including the factor of (1/2) from the Jacobian gives:

$$I = \frac{2}{2}\int (L + \tau)\, F(\tau)\, d\tau + \frac{2}{2}\int (L - \tau)\, F(\tau)\, d\tau$$

$$I = \int_{-L}^{L} L\, F(\tau)\, d\tau + \int_{-L}^{o} \tau\, F(\tau)\, d\tau - \int_{o}^{L} \tau\, F(\tau)\, d\tau . \qquad \text{(B4)}$$

Because $F(\tau)$ is an even function, we can write:

$$\int_{-L}^{o} \tau\, F(\tau)\, d\tau = -\int_{o}^{L} \tau F(\tau)\, d\tau$$

and (B5)

$$\int_{-L}^{L} L\, F(\tau)\, d\tau = 2\int_{o}^{L} LF(\tau)\, d\tau .$$

The total integral can then be written as:

$$I = 2\int_{o}^{L} F(\tau)\, (L-\tau)\, d\tau . \qquad \text{(B6)}$$

The double integral for the mean scattered power can then be written as:

$$\sigma^{o} = \frac{\pi F^{2}}{L\lambda}\int_{o}^{L} \left|\chi_2 - \chi_1 \chi_1^{*}\right| \cos\left(v_x \tau\right) (L-\tau)\, d\tau . \qquad \text{(B7)}$$

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